9.1 Functions and Operations

You have seen that two functions, $f(x)$ and $g(x)$, can be added, subtracted, multiplied, or composed to produce a third function.

- Adding $f(x)$ and $g(x)$ results in a new function called $(f + g)(x)$.
- Subtracting $f(x)$ and $g(x)$ results in a new function called $(f - g)(x)$.
- Multiplying $f(x)$ and $g(x)$ results in a new function called $(f \bullet g)(x)$.
- $f(x)$ and $g(x)$ can be composed in two ways: $f(g(x))$ and $g(f(x))$.

The value of a function for a given x-value is the vertical height or the second coordinate of the point $(x, f(x))$. The vertical heights can be used to perform operations on functions.

Example 1

If $f(x) = 4x - 3$ and $g(x) = x + 2$, find $(f \bullet g)(3)$. Check your answer.

Solution

Method 1

Evaluate $f(3)$ and $g(3)$ separately and multiply the results.

$$\begin{aligned}(f \cdot g)(3) &= f(3) \cdot g(3) \\ &= [4(3) - 3][3 + 2] \\ &= (12 - 3)(5) \\ &= 9(5) \\ &= 45\end{aligned}$$

Method 2

Find an expression for $f(x) \bullet g(x)$ and evaluate for $x = 3$.

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x) \\ &= (4x - 3)(x + 2) \\ &= 4x^2 - 3x + 8x - 6 \\ &= 4x^2 + 5x - 6\end{aligned}$$

$$\begin{aligned}4x^2 + 5x - 6 &= 4(3)^2 + 5(3) - 6 \\ &= 4(9) + 15 - 6 \\ &= 36 + 15 - 6 \\ &= 51 - 6 \\ &= 45\end{aligned}$$

Example 2

If $f(x) = 2x + 1$ and $g(x) = 3x^2$, find an expression for $g(f(x))$.

Solution

$$\begin{aligned}g(f(x)) &= g(2x + 1) && \text{Substitute } 2x + 1 \text{ for } f(x). \\ &= 3(2x + 1)^2 && \text{Find } g(2x + 1). \\ &= 3(4x^2 + 4x + 1) \\ &= 12x^2 + 12x + 3\end{aligned}$$