Expressions and Functions

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Objective

The differences between expressions and functions are sometimes blurred. In this project we try to clarify these differences since expressions and functions are treated very differently in Maple.

Narrative

An *expression* is a quantity formed by using mathematical operations (such as addition, subtraction, multiplication, and division) to combine constants and variables. For example, $2x + 3$ is an expression. A *function* is a rule of correspondence that associates to each element in one set (the domain of the function) an element in another set (the range of the function). For example, $f(x) = 2x + 3$ is a function. A function such as $f(x) = 2x+3$ is sometimes written (by hand) as $f : x \to 2x+3$ and sometimes simply as $x \to 2x+3$.

The following examples are intended to clarify the differences between expressions and functions:

- x^2 and y^2 are two different expressions (since they involve two different variables), but $x \to x^2$ and $y \to y^2$ are the same function (since — however you choose to write it — the "rule" being described here is, "square your favorite value").
- $\dfrac{x^2-1}{x-1}$ and $x + 1$ are the same expressions (that is, $\dfrac{x^2-1}{x-1} = x + 1$), but $x \to \dfrac{x^2-1}{x-1}$ and $x \to x + 1$ are different functions (since the former function is not defined when $x = 1$ but the later is).

Again, an expression is a *quantity* and a function is a *rule*. With expressions, the names of variables are important; with functions $f(x)$ the variable x is just a place-holder that makes it easier to write the rule that defines the function. With functions, the rule that specifies how the function is to be computed is important, but with expressions there isn't any "rule of correspondence" like there is for functions. (You can substitute values into an expression and arrive at a numerical value, but, as the second example above illustrates, if you substitute the same value into two different forms for the same expression then you might not get the same thing.)

Tasks

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. The effect of each command is described in the right-hand column for your reference. These commands define an expression, substitute a value for x into this expression, define a function, and evaluate the function for a specific value of the variable x.

`> # Your name, today´s date`	
`> # Expressions and Functions`	
`> restart;`	
`> # Task 1`	
`> expr := 2*x+3;`	Let *expr* be the expression $2x + 3$.
`> subs(x=1,expr);`	Substitute 1 for x in $expr = 2x + 3$.
`> f := x -> 3*x-2;`	Let f be the function $f := x \to 3x - 2$.
`> f(2);`	Evaluate f when $x = 2$.

2. Continue by typing the following commands into Maple in the order listed. With these commands we illustrate that if f is a function then $f(x)$ is an expression in the variable x, and we can substitute a value for x into this expression. We also illustrate that if *expr* is an expression in x then the Maple command *unapply* can be used to identify the function g for which $g(x) = expr$.

```
> # Task 2
> subs(x=2,f(x));
```
Substitute 2 for x in the expression $f(x)$.
(The value Maple returns should be $f(2)$.)

```
> g := unapply(expr,x);
```
Let g be the function defined by the expression *expr*.

```
> g(1);
```
Evaluate g when $x = 1$.
(The value Maple returns should be the value you obtained by substituting 1 for x in *expr*.)

3. Continue by typing the following commands into Maple in the order listed. To appreciate the significance of these commands lines it's important to know that the first argument of the `plot` command is an expression (not a function: note the error message in the last line where we try to plot a function).

```
> # Task 3
> plot(expr,x=0..2);
> plot(f(x),x=0..2);
> plot(f,x=0..1);
```

At this time make a hard-copy of your typed input and Maple's responses. Then:

4. In writing:

a) Define the terms "expression" and "function", and give an example of each.

b) Without reference to Maple, and in your own words, describe the differences between expressions and functions.

Your lab report will be a hard-copy of your typed input and Maple's responses (both text and hand-labeled graphics), and your written responses.

Comments

Later we will discuss a process known as differentiation, and Maple distinguishes between differentiation of expressions and functions: `diff(expr,x)` is the derivative of the expression *expr* with respect to x, and `D(f)` is the derivative if the function x (which is a function whose value at `x` is `D(f)(x)`.

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