A Brief Maple Tutorial
Part 2: Functions of a Single Variable and Graphing

Michael Penna, Indiana University – Purdue University, Indianapolis

Objective

In this project we discuss how to define and graph functions in Maple.

Narrative

In Maple, the function $f = f(x)$ is defined by the command:

`f := x ->` <expression in `x`>

and:

`plot(f(x),x=a..b)`	plots f from $x = a$ to $x = b$,
`plot(f(x),x=a..b,y=c..d)`	plots f from $x = a$ to $x = b$, limiting output to points with y-coordinates between c and d,
`plot({f(x),g(x)},x=a..b,y=c..d)`	plots the functions f and g from $x = a$ to $x = b$, restricting output to points with y-coordinates between c and d.

Task

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. The effect of each command is described in the right-hand column for your reference, along with some questions you should ask yourself.

`> # Your name, today´s date`	
`> # Functions of a Single Variable and Graphing`	
`> # Task 1`	
`> restart;`	Clear Maple's memory.
`> f := x -> sin(x);`	Let $f(x) = \sin x$. (See Comment 1 below.)
`> f(Pi/2);`	Evaluate f at $x = \pi/2$. (Note that trig function arguments are measured in radians in Maple.)
`> f(3);`	Evaluate f at $x = 3$.
`> plot(f(x),x=-100..100);`	Plot f from $x = -100$ to $x = 100$. (What does this graph tell you about the behavior of f?)
`> plot(f(x),x=-1..1);`	Plot f from $x = -1$ to $x = 1$. (What does *this* graph tell you about the behavior of f? It's not so clear anymore, is it?! How can you reconcile this graph with your earlier one? Well, ...)
`> plot(f(x),x=-4..4);`	Plot $f(x)$ from $x = -4$ to $x = 4$.

The point we're trying to make here is that the choice of x-values makes a big difference in getting a good plot of the graph of a function. The following command lines illustrate that the same thing is true for the choice of the y-values. Continue by typing them into Maple in the order in which they are listed.

`> plot(tan(x),x=-Pi..Pi);`	Plot $\tan(x)$ from $x = -\pi$ to $x = \pi$. (Does this look like the graph of $\tan x$?)
`> plot(tan(x),x=-Pi..Pi,y=-3..3);`	Plot $\tan(x)$ from $x = -\pi$ to $x = \pi$, restricting output to points with y-coordinates between -3 and 3. (We get a much better graph by restricting y-coordinates, but now we have different scales on the x- and y-axes.)

In summary, Maple is very useful for graphing, but you cannot trust it completely: it might *not* reveal important behavior of a function or provide a good picture of a function if you look at a graph of the function at the wrong scale.

2. Continue by typing the command lines below into Maple. The second line illustrates how we can plot more than one function in one graphic.

```
> # Task 2
> plot({x,2*sin(x)},x=-Pi..Pi);
```

Graph $f(x) = x$ and $g(x) = 2\sin x$ from $x = -\pi$ to $x = \pi$ in one graphic.

We now find the coordinates of a point $P(x, y)$ of intersection of these graphs, in effect finding one of the solutions of the equation $x = 2\sin x$. Looking at our graph we observe that one solution lies in the interval $[1.8, 2.0]$; so we graph both functions over this interval: continue by typing the following command line into Maple.

```
> plot({x,2*sin(x)},x=1.8..2.0);
```

Plot the graphs of $f(x) = x$ and $g(x) = 2\sin x$ from $x = 1.8$ to $x = 2.0$.

From this graph it appears that solution lies in the interval $[1.89, 1.90]$; so we graph both functions over *this* interval: continue by typing the following command line into Maple.

```
> plot({x,2*sin(x)},x=1.89..1.90);
```

Replot $f(x)$ and $g(x)$ from $x = 1.89$ to $x = 1.90$.

From this graph it appears that the solution lies in the interval $[1.895, 1.896]$. By continuing the process of "zooming in", we could (theoretically, at least) arrive at an arbitrarily precise estimate of the x-value for which $f(x) = g(x)$ or $x = 2\sin x$.

Zoom in one more time to find two numbers, to four decimal places each, between the solution lies. State these values. (A comment of the form, "The correct value of x lies between ... and ...." will suffice.)

At this time, make a hard-copy of your typed input and Maple's responses. Then, in each of the last four graphics, label by hand the graphs of $f(x) = x$ and $g(x) = 2\sin x$ by "$f(x) = x$" and "$g(x) = 2\sin x$", respectively.

Your lab report will be a hard copy of your typed input and Maple's responses (both text and hand-labeled graphics).

Comments

1. Note that when using a function such as $\sin x$ in Maple you must type "`sin(x)`" (being careful to include parentheses) rather than "`sin x`" (which you might write by hand).

2. Maple allows you to control numerous aspects of the appearance of graphics via "options". For example, to draw the graph of the function f from $x = a$ to $x = b$ in green (rather than in the default color of red), you would use the command

   ```
   plot(f(x),x=a..b,color=green)
   ```

   While there are many advantages to using different colors when viewing a graphic, a potential disadvantage to using certain colors is that they don't always come out well when you print them. Since this issue is hardware dependent, try the code in these projects as written, but if your graphics don't come out well when you print them, try an option such as "`color=black`" to get a better hard copy.

3. Maple has several "packages" of functions which, while not automatically loaded when you start a Maple session (to conserve on memory), are easy to load and very useful. For example, one of Maple's packages is the `plots` package; you load the `plots` package (or any package, for that matter) with the `uses` command:

```
> uses(plots);
```

And one of the useful functions in the `plots` package is the `setoptions` command: If you want to use the same options throughout a Maple session, you can specify them once at the beginning of the session using `setoptions`, and ignore them thereafter (saving time while reducing both typing and subsequent command length). For example, if we had included the command lines

```
> uses(plots);
> setoptions(color=blue);
```

immediately after the `restart` command, then all subsequent plots would have (by default) included the option "`color=blue`".

4. An option we can use in drawing the graph of a function with vertical asymptotes is `discont`. In drawing the graph of the tangent function, for example, vertical asymptotes appeared at odd multiples of $\pi/2$ in our output. If we had used "`plot(tan(x), x=-Pi..Pi, y=-3..3, discont=true)`" then they would not have appeared.

5. Another way to solve the equation $x = 2\sin x$ graphically is by "zooming in" on the zero of the function $h(x) = x - 2\sin x$.

6. Observe that the more you "zoom in" on the graph of $g(x) = 2\sin x$, the more linear it appears. Do you think this is true for all functions?

7. Even though the graphs of curves may appear to be smooth, the way programs like Maple graph functions is by plotting a finite number of points and "connecting-the-dots" with short line segments. Thus, rather than seeing a smooth curve when you plot a function such as $\sin x$, you are actually seeing a polygonal approximation to its graph. (To see the dots that Maple uses to graph f, enter the command "`plot(f(x),x=a..b,style=point)`".)

8. To specify a piecewise-defined function such as

$$f(x) = \begin{cases} x^2 & \text{if } x \le 0 \\ 2x+1 & \text{if } 0 < x \le 1 \\ 4 & \text{if } x > 1 \end{cases}$$

in Maple you may use:

```
f := x -> if x <= 0 then x^2 elif x <= 1 then 2*x+1 else 4 end if;
```

In addition to the "`if/then/elif/else/end`" control structure, Maple offers many other structures; to learn more about them, check out Maple's Help. You may also use the `piecewise` command to specify a piecewise-defined function: to use this command to define the function f above, you could write

```
f := x -> piecewise(x<=0, x^2, x>0 and x<=1, 2*x+1, x>1, 4);
```

To learn more about the `piecewise` command, check out Maple's Help.