Calculus, Maple, and Complex Numbers

Michael Penna, Indiana University – Purdue University, Indianapolis

Objective

To discuss how complex numbers arise in the study of Calculus using Maple.

Narrative

While we will focus almost exclusively on real numbers in this sequence of first courses in Calculus, Maple's default is to perform computations using complex numbers. In this project we illustrate how Maple's use of complex numbers can affect even simple computations and graphics.

Task

1. Type the command lines below into Maple in the order in which they are listed. The effect of each command is described in the right-hand column for your reference.

`> # Your name, today´s date`	
`> # Calculus, Maple, and Complex Numbers`	
`> # Task 1`	
`> restart;`	Clear Maple's memory.
`> 8^(1/3);`	Try to compute $8^{1/3}$.
`> evalf(%);`	Write $8^{1/3}$ in floating point notation.
`> (-8)^(1/3);`	Try to compute $(-8)^{1/3}$.
`> evalf(%);`	Write $(-8)^{1/3}$ in floating point notation.

Observe that Maple thinks of $(-8)^{1/3}$ as a complex number!

2. Continue by typing the command lines below into Maple.

`> # Task 2`	
`> f := x -> x^(1/3);`	Let $f(x) = x^{1/3}$.
`> plot(f(x),x=-1..1);`	Plot the graph of f over the interval $[-1, 1]$.

You may observe a problem with the resulting graph: the part that corresponds to $x < -1$ is missing! To address this problem, try typing the following command lines into Maple.

`> f := x -> surd(x,3);`	See the narrative below.
`> plot(f(x),x=-1..1);`	Plot the graph of f over the interval $[-1, 1]$.

To understand the above input and Maple's output, observe that there is an inconsistency in the way we often do real arithmetic:

$$(-8)^{1/3} = -2 \quad \text{but} \quad (-8)^{2/6} = ((-8)^2)^{1/6} = 64^{1/6} = 2.$$

To account for this inconsistency Maple returns a real value for $a^{1/3}$ if $a > 0$, and a complex value for $a^{1/3}$ if $a < 0$. And when we try to `plot` the function "`f := x -> x^(1/3)`" in Maple we only see a graph for values of $x \geq 0$. To obtain the correct (real) graph for f in Maple — a graph that is correct for both $x \geq 0$ and $x \leq 0$ — we use the `surd` command:

$$\texttt{surd(x,n)} = \begin{cases} x^{1/n} & \text{if } n \text{ is odd and } x > 0 \\ -(-x)^{1/n} & \text{if } n \text{ is odd and } x < 0. \end{cases}$$

To emphasize this point, continue by typing the command lines below into Maple in the order in which they are listed.

`> f := x -> x*(1-x^2)^(2/3);`	Let $f(x) = x(1-x^2)^{2/3}$.
`> plot(f(x),x=-2..2);`	Plot the graph of f over the interval $[-2, 2]$.
`> f := x -> x*surd(1-x^2,3)^2;`	Let $f(x) = x(1-x^2)^{2/3}$.
`> plot(f(x),x=-2..2);`	Plot the graph of f over the interval $[-2, 2]$.

At this time make a hard-copy of your typed input and Maple's responses. Then:

3. By hand, label the first graphic you produced in Task 2, "$f(x) = x^{1/3}$" and highlight that part of the x-axis for which $f(x)$ returns real values. Then, again by hand, label the second graphic you produced in Task 2, "$f(x) = \text{surd}(x, 3)$" and highlight that part of the x-axis for which that $f(x)$ returns real values. Repeat this process with the third and fourth graphics you produced in Task 2.

Your lab report will be a hard copy of your typed input and Maple's responses (both text and hand-labeled graphics).