One-Sided Limits

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Objective

To investigate the one-sided limits of a function at a point, numerically.

Narrative

Prior to having the theorems on one-sided limits at our disposal, there are two major issues surrounding the one-sided limits of a function at a point: The first is guessing what they are, if they even exist; this issue can often be approached either graphically or numerically. The second issue involves proving that the guesses you made are correct; this issue involves using the formal definition of one-sided limit.

In this project we address the issue of guessing one-sided limits numerically. In this project we also illustrate how to perform repeated computations efficiently in Maple using a "do loop".

Tasks

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands will help you estimate the one-sided limits and the limit (if either or both exist) of $f(x) = x^2 + [\![x]\!]$ at $x = 1$. (*Notes*: It's OK to type either of the loops "`for n from 1 to 6 do ... end do:`" on one line. And the greatest integer function $[\![x]\!]$ is "`trunc(x)`" in Maple.)

`> # Your name, today´s date`	
`> # One-Sided Limits`	
`> restart;`	Clear Maple's memory.
`> f := x -> x^2+trunc(x);`	Let $f(x) = x^2 + [\![x]\!]$.
`> plot(f(x),x=-2..2,discont=true);`	Plot the graph of f.
`> a := 1.0;`	Let $a = 1$.
`> f(a);`	What is $f(a)$?
`> for n from 1 to 6`	Let's look at the values of $f(x)$ for $x < a$.
`do`	This is the beginning of a "do loop".
`x := a-1/2^n:`	Let $x = a - \frac{1}{2^n}$.
`print(evalf(x), evalf(f(x)));`	Print the values of x and $f(x)$.
`end do:`	This is the end of the "do loop".
`> for n from 1 to 6`	Now let's look at some values of $f(x)$ for $x > a$.
`do`	
`x := a+1/2^n:`	
`print(evalf(x), evalf(f(x)));`	
`end do:`	

At this time make a hard-copy of your typed input and Maple's responses. Then:

2. On the graphic you created in Task 1, plot the points on the graph of f whose values you computed in the two "do loops". (The easiest way to do this is to plot x values along the x-axis and then draw lines parallel to the y-axis through these points, until they meet the graph of f.)

3. On the basis of this data, do you think $\lim_{x\to a^+} f(x)$ exists? If so, what is it (to 4 decimal places of accuracy)? Do you think $\lim_{x\to a^-} f(x)$ exists? If so, what is it (to 4 decimal places of accuracy)? Do you think $\lim_{x\to a} f(x)$ exists? If so, what is it (to 4 decimal places of accuracy)? If you think one or more of these limits does not exist, justify your answer

Your lab report will be a hard copy of your typed input and Maple's responses (both text and hand-labeled graphics).

Comments

Maple has built in commands that allow you to compute one-sided limits automatically. Since we are interested not just in what limits are, but how they are computed, we intentionally avoided using this command in this project.