Graphing a Function and its Derivative

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Objective

To review the graphing of functions by plotting points and "connecting the dots", and to investigate the connection between f and f' as functions.

Narrative

Recall that since $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$, for small values of h,

$$f'(x) \approx \frac{f(x+h)-f(x)}{h}. \qquad (*)$$

Hence f is increasing if and only if $f'(x) > 0$, and f is decreasing if and only if $f'(x) < 0$.

Task

1. Type the command lines below into Maple in the order in which they are listed. These commands initialize Maple and define the procedure `pointset` which associates to a function `g`, to two real numbers `a` and `b`, and to an integer `N`, a list containing the coordinates of `N` points on the graph of `g` from `a` to `b`.

```
> # Your name, today´s date
> # Graphing a Function and its Derivative
> # Task 1
> restart;
> with(plots):
> pointset := proc(g,a,b,N)
   local mylist,n,dx:
   dx := (b-a)/N:
   mylist := [a,evalf(g(a))]:
   for n from 1 to N-1 do
    mylist := mylist,[a+n*dx,g(a+n*dx)]:
   end do:
   RETURN([mylist]);
  end:
```

2. Continue by typing the command lines below into Maple in the order in which they are listed. These commands produce a plot of several points on the graphs of $f(x) = (1-x^2)/(1+x^2)$ and $f'(x), x \in [-2,2]$.

```
> # Task 2
> f := x -> (1-x^2)/(1+x^2);
> f1 := D(f);
> a := -2.0; b := 2.0; N := 20;
> plot0 := pointplot(pointset(f,a,b,N),color=red,symbol=point):
> plot1 := pointplot(pointset(f1,a,b,N),color=green,symbol=point):
> display({plot0,plot1},scaling=constrained);
```

3. Continue by typing the command lines below into Maple in the order in which they are listed. These commands produce a table of values for $f(x) = (1-x^2)/(1+x^2)$ and $f'(x)$, $x \in [-2,2]$.

```
> # Task 3
> M := matrix(N+1,3,(row,col)->0):
> M[1,1] := ´x´; M[1,2] := ´f(x)´; M[1,3] := ´f1(x)´;
> Digits := 2;
> dx := (b-a)/N;
> for n from 1 to N do
   x := a + (n-1)*dx:
   M[n+1,1] := x:  M[n+1,2] := f(x):  M[n+1,3] := f1(x):
   end do:
> eval(M);
```

At this time make a hard-copy of your typed input and Maple's responses. Then:

3. On the graphic you produced in Task 2 graph f and f' by hand (by "connecting the dots"), each in a different color, and label each graph.

4. Illustrate (*) by using the table of values you produced in Task 3 to:

a) approximate $f'(1.2)$ using values in the *second* column of the table, and

b) compute the absolute value of the difference between this approximation and the actual value of $f'(1.2)$ (which you can obtain from the *third* column of the table). This is the *error* in the approximation.

Your lab report will be a hard-copy of your typed input and Maple's responses (both text and hand-labeled graphics), and your written responses.