

# Maximum and Minimum Values on a Closed Interval

Michael Penna, Indiana University – Purdue University, Indianapolis

## Objective

To illustrate how to find the maximum and minimum values of a function on a closed interval.

## Narrative

The critical numbers of  $f$  on the interval  $(a, b)$  are the values of  $x \in (a, b)$  for which  $f'(x) = 0$  and  $f'(x)$  does not exist. To find the absolute maximum and minimum values of a function  $f$  on a closed interval  $[a, b]$  we first find the critical numbers of  $f$  on the open interval  $(a, b)$ , and then compare the values of  $f$  at these points with the values  $f(a)$  and  $f(b)$  of  $f$  at the endpoints. In this project we illustrate this process as we find the absolute maximum and minimum values of  $f(x) = 2 + (x + 1)^{2/3}$  on the interval  $[-2, 7]$ .

## Tasks

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. The effect of each command is described in the right-hand column for your reference.

> # Your name, today's date	
> # Maximum and Minimum Values on a Closed Interval	
> # Task 1	
> restart;	Clear Maple's memory.
> Digits := 2;	
> f := x -> 2+surd(x+1,3)^2;	Let $f(x) = 2 + (x + 1)^{2/3}$ .
> f1 := D(f);	Let f1 denote the first derivative $f'$ of $f$ .
> f1exp := simplify(D(f)(x));	Let f1exp denote the simplified expression $f'(x)$ .
> evalf(solve(numer(f1exp)=0,x));	Find where the numerator of $f'(x)$ is 0.
> evalf(solve(denom(f1exp)=0,x));	Find where the denominator of $f'(x)$ is 0.

2. Use Maple to evaluate  $f$  at each critical number as well as at the endpoints  $-2$  and  $7$  of the interval  $[-2, 7]$ .

3. Use Maple to draw the graph of  $f$  over the interval  $[-2, 7]$ .

At this time make a hard-copy of your typed input and Maple's responses. Then:

4. By hand, plot and label the points  $P(c, f(c))$  where  $c$  is a critical number of  $f$ , as well as the points  $A(a, f(a))$  and  $B(b, f(b))$ , on the graphic you created in Task 3.

5. Underneath your labeled graphic, state the maximum and minimum values of  $f$  on  $[-2, 7]$ . (A statement of the form, “\_\_\_ is the maximum value for  $f$ , and \_\_\_ is the minimum value for  $f$ .” will suffice.)

Your lab report will be a hard-copy of your typed input and Maple's responses (both text and hand-labeled graphics).