# Finding Relative Extrema on an Open Interval: The First Derivative Test

*Michael Penna, Indiana University – Purdue University, Indianapolis*

### *Objective*

To illustrate how to find and classify the relative extrema of a function on an open interval using the First Derivative Test.

### *Narrative*

Recall that to find the maximum and minimum values of any function $f(x)$ on an open interval using the First Derivative Test we:

1. find the critical numbers of $f$ (the values of $x$ for which $f'(x) = 0$ and $f'(x)$ does not exist),
2. compute the sign of the first derivative $f'$ of $f$ at points $x_0$ and $x_1$ just to the left and right of each critical number $c$ ($x_0 < c < x_1$): if
   (a) $f'(x_0) > 0$ and $f'(x_1) < 0$ then $f(c)$ is a relative maximum,
   (b) $f'(x_0) < 0$ and $f'(x_1) > 0$ then $f(c)$ is a relative minimum,
   (c) $f'(x_0) > 0$ and $f'(x_1) > 0$, or $f'(x_0) < 0$ and $f'(x_1) < 0$, then $f(c)$ is a neither a relative maximum nor a relative minimum. (In this last case, the point $P(c, f(c))$ is a saddle point.)

In this project we illustrate this process as we find the relative maximum and minimum values of $f(x) = x(1 - x^2)^{2/3}$.

### *Tasks*

**1**. a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. The effect of each command is described in the right-hand column for your reference.

| | |
|---|---|
| `> # Your name, today´s date` | |
| `> # The First Derivative Test` | |
| `> restart;` | Clear Maple's memory. |
| `> Digits := 5;` | |
| `> f := x -> x*surd(1-x^2,3)^2;` | Let $f(x) = x(1 - x^2)^{2/3}$. |
| `> f1 := D(f);` | Let `f1` denote the first derivative $f'$ of $f$. |
| `> f1exp := simplify(D(f)(x));` | Let `f1exp` denote the simplified expression $f'(x)$. |
| `> evalf(solve(numer(f1exp)=0,x));` | Find where the numerator of $f'(x)$ is 0. |
| `> evalf(solve(denom(f1exp)=0,x));` | Find where the denominator of $f'(x)$ is 0. |

b) Use Maple to evaluate $f'$ (or `f1`) at points $x_0$ and $x_1$ just to the left and right of each critical number.

c) On the basis of your computations, write Maple comments stating whether $f(c)$ is a relative maximun or relative minimum, or whether $P(c, f(c))$ is a saddle point, for each critical number $c$. (Your comment might look like, "`# __ is a relative maximum`" for example.)

d) Type the following command line into Maple, substituting for `a`, `b`, `c` and `d` numbers that are large enough to enclose all the critical numbers of $f$ and the graph of $f$ over $[a, b]$.

| | |
|---|---|
| `> plot(f(x),x=a..b,y=c..d);` | Plot the graph of $f$ over the interval $[a, b]$. |

At this time make a hard-copy of your typed input and Maple's responses. Then:

**2**. By hand, on the graphic you produced in Task 1, label the graph of $f$, plot the critical numbers of $f$ along the $x$-axis, and plot and label the points $P(c, f(c))$, $c$ a critical number of $f$. Remembering that the relative extrema of $f$ are $y$-values, estimate the relative extrema of $f$ by eye and for each relative extremum write a sentence under your graphic of the form, “$f$ has a relative maximum of _ when $x =$ _” or “$f$ has a relative minimum of _ when $x =$ _”.

Your lab report will be a hard-copy of your typed input and Maple’s responses (both text and hand-labeled graphics).