

The Second Derivative and Concavity

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Objective

To illustrate the connection between the second derivative f'' of a function f and the concavity of the graph of f .

Narrative

Recall that the graph of a function f is concave up at x if $f''(x) > 0$ and concave down at x if $f''(x) < 0$. In this project we illustrate this connection for $f(x) = x^4 - 4x^3$.

Task

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. The effect of each command is described in the right-hand column for your reference.

> # Your name, today's date	
> # The Second Derivative and Concavity	
> restart;	Clear Maple's memory.
> f := x -> x^4-4*x^3;	Let $f(x) = x^4 - 4x^3$.
> f1 := D(f);	Let f1 denote the first derivative f' of f .
> f2 := D(f1);	Let f2 denote the second derivative f'' of f .
> plot({f(x),f2(x)},x=-1..4);	Plot the graphs of f and f'' over $[-1, 4]$.

At this time make a hard-copy of your typed input and Maple's responses. Then:

2. By hand, label the graphs of f and f'' in the graphic you produced in Task 1.
3. By hand, highlight that part of the graph of f over which f is concave up and that part of the graph of f'' over which f'' is positive.

Your lab report will be a hard copy of your typed input and Maple's responses (both text and graphics).