

# Concavity and Inflection Points

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## Objective

To illustrate how to analyze the concavity and find the inflection points of a function.

## Narrative

The graph of a function  $f$  is concave up at  $x$  if  $f''(x) > 0$  and concave down at  $x$  if  $f''(x) < 0$ , and an *inflection point* of  $f$  is a point  $P(x, f(x))$  on the graph of  $f$  at which the concavity of  $f$  changes. To find the inflection points of  $f$ , we first find the values of  $x$  for which  $f''(x) = 0$  and for which  $f''(x)$  does not exist; these are the *potential inflection numbers* of  $f$ . We then look at the value of the second derivative just to the left and just to the right of each potential inflection number  $x$ : if these values have different signs then  $x$  is an *inflection number*, and  $P(x, f(x))$  is an *inflection point* of  $f$ ; if these values have the same sign then  $x$  is not an inflection number, and  $P(x, f(x))$  is not an inflection point of  $f$ .

In this project we analyze the concavity and find the inflection points of  $f(x) = x^5 - 5x^3$ .

## Task

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. The effect of each command is described in the right-hand column for your reference.

> # Your name, today's date	
> # Concavity and Inflection Points	
> restart;	Clear Maple's memory.
> f := x -> x^5-5*x^3;	Let $f(x) = x^5 - 5x^3$ .
> f1 := D(f);	Let $f1$ denote the first derivative $f'$ of $f$ .
> f2 := D(f1);	Let $f2$ denote the second derivative $f''$ of $f$ .
> solve(f2(x)=0,x);	Find where $f''(x) = 0$ .
> fsolve(f2(x)=0,x);	Find where $f''(x) = 0$ numerically.

b) Use Maple to evaluate  $f''$  (or  $f2$ ) at points  $x_0$  and  $x_1$  just to the left and right of each potential inflection number of  $f$ .

c) On the basis of your computations, write Maple comments stating whether  $P(c, f(c))$  is an inflection point, for each potential inflection number  $c$ . (Your comment might look like, “# \_\_\_\_\_ is an inflection point” for example.)

d) Type the following command line into Maple, substituting for  $a$ ,  $b$ ,  $c$  and  $d$  numbers that are large enough to enclose all the potential inflection numbers of  $f$  and the graphs of  $f$  and  $f''$  over  $[a, b]$ .

> plot(f(x),x=a..b,y=c..d);	Plot the graph of $f$ over an $x$ -range large enough to capture all relevant function behavior.
> plot({f(x),f2(x)},x=a..b,y=c..d);	Plot the graphs of $f$ and $f''$ over an $x$ -range large enough to capture all relevant function behavior.

At this time make a hard-copy of your typed input and Maple's responses. Then:

2. By hand, on the first graphic you produced in Task 1, label the graph of  $f$ , plot the potential inflection numbers of  $f$  along the  $x$ -axis, and plot and label the points  $P(c, f(c))$ ,  $c$  a potential inflection number of  $f$ . For each inflection point write a sentence under your graphic of the form, “\_\_\_\_\_ is an inflection point”.

3. By hand, on the second graphic you produced in Task 1, label the graphs of  $f$  and  $f''$ . Then highlight that part of the graph of  $f$  which is concave up, and that part of the graph of  $f''$  over which  $f''$  is positive.

Your lab report will be a hard-copy of your typed input and Maple's responses (both text and hand-labeled graphics).

### ***Comments***

An inflection point is a point on the graph of a function at which the concavity changes, and we can narrow our search for inflection points to a finite number of points by finding where  $f''(x) = 0$  and where  $f''(x)$  does not exist. We restricted our attention to points at which  $f''(x) = 0$  in the above discussion and example since  $f(x)$  is a polynomial function so  $f''(x)$  exists for all  $x$ .