

Concavity and Inflection Points

Michael Penna, Indiana University – Purdue University, Indianapolis

Objective

To illustrate how to analyze the concavity and find the inflection points of a function.

Narrative

The graph of a function f is concave up at x if $f''(x) > 0$ and concave down at x if $f''(x) < 0$, and an *inflection point* of f is a point $P(x, f(x))$ on the graph of f at which the concavity of f changes. To find the inflection points of f , we first find the values of x for which $f''(x) = 0$ and for which $f''(x)$ does not exist; these are the *potential inflection numbers* of f . We then look at the value of the second derivative just to the left and just to the right of each potential inflection number x : if these values have different signs then x is an *inflection number*, and $P(x, f(x))$ is an *inflection point* of f ; if these values have the same sign then x is not an inflection number, and $P(x, f(x))$ is not an inflection point of f .

In this project we analyze the concavity and find the inflection points of $f(x) = x(1 - x^2)^{1/3}$.

Task

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. The effect of each command is described in the right-hand column for your reference.

> # Your name, today's date	
> # Concavity and Inflection Points	
> restart;	Clear Maple's memory.
> Digits := 5;	
> f := x -> x*surd(1-x^2,3);	Let $f(x) = x(1 - x^2)^{1/3}$.
> f1 := D(f);	Let f1 denote the first derivative f' of f .
> f2 := D(f1);	Let f2 denote the second derivative f'' of f .
> f2exp := simplify(f2(x));	Let f2exp denote the simplified f2.
> evalf(solve(numer(f2exp)=0,x));	Find where the numerator of f2exp is 0.
> evalf(solve(denom(f2exp)=0,x));	Find where the denominator of f2exp is 0.

b) Use Maple to evaluate f'' (or f2) at points x_0 and x_1 just to the left and right of each potential inflection number of f .

c) On the basis of your computations, write Maple comments stating whether $P(c, f(c))$ is an inflection point, for each potential inflection number c . (Your comment might look like, “# _____ is an inflection point” for example.)

d) Type the following command line into Maple, substituting for a, b, c and d numbers that are large enough to enclose all the potential inflection numbers of f and the graphs of f and f'' over $[a, b]$.

> plot(f(x),x=a..b,y=c..d);	Plot the graph of f over an x -range large enough to capture all relevant function behavior.
> plot({f(x),f2(x)},x=a..b,y=c..d);	Plot the graphs of f and f'' over an x -range large enough to capture all relevant function behavior.

At this time make a hard-copy of your typed input and Maple's responses. Then:

2. By hand, on the first graphic you produced in Task 1, label the graph of f , plot the potential inflection numbers of f along the x -axis, and plot and label the points $P(c, f(c))$, c a potential inflection number of f . For each inflection point write a sentence under your graphic of the form, “_____ is an inflection point”.

3. By hand, on the second graphic you produced in Task 1, label the graphs of f and f'' . Then highlight that part of the graph of f which is concave up, and that part of the graph of f'' over which f'' is positive.

Your lab report will be a hard-copy of your typed input and Maple's responses (both text and hand-labeled graphics).