

Graphing Functions

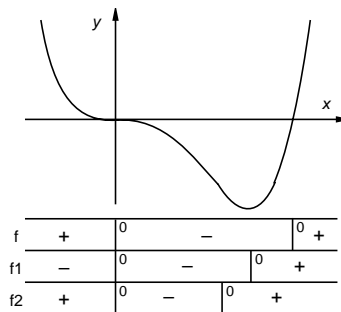
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Objective

To investigate what we can say about the graph of a function f given where f and its first and second derivatives are positive, negative, and zero.

Narrative

To record where a function f and its first and second derivatives f' and f'' , respectively, are positive, negative, and zero, we use three “recording strips” below the graph of f as illustrated in the figure at the right. Study this figure carefully. Note how the vertical bars are drawn where f , f' , and f'' are 0, and the spaces between the vertical bars are labelled + or – depending on the sign of f , f' , and f'' over those intervals.



Tasks

1. Type the command lines below into Maple in the order in which they are listed. They produce a graph of the function $f(x) = 4x/(1+x^2)$, and three recording strips below the graph of f .

```
> # Your name, today's date
> # Graphing Functions
> restart;
> with(plots):
> # Task 1
> f := x -> 4*x/(1+x^2);
> plot0 := plot({-6,-5,-4,-3,0},x=-6..6,y=-6..3,color=black):
> plot1 := textplot({[-6,-3.5,'f'],[-6,-4.5,'f1'],[-6,-5.5,'f2']});
> plot2 := plot(f(x),x=-6..6):
> display({plot0,plot1,plot2});
```

2. a) Type the command lines below into Maple in the order in which they are listed. They determine where $f(x) = x^2(1-x)^{2/3}$ and its first and second derivatives are zero; this information will be used later in this project.

```
> # Task 2
> Digits := 5;
> f := x -> x^2*surd(1-x,3)^2;
> evalf(solve(f(x)=0,x));
> f1 := D(f);
> f1exp := simplify(f1(x));
> evalf(solve(numer(f1exp)=0,x));
> evalf(solve(denom(f1exp)=0,x));
> f2 := D(f1);
> f2exp := simplify(f2(x));
> evalf(solve(numer(f2exp)=0,x));
> evalf(solve(denom(f2exp)=0,x));
```

b) Type the command line below into Maple. It produces an empty graph and three recording strips.

```
> display({plot0,plot1});
```

At this time make a hard-copy of your typed input and Maple's responses. (But don't shut down Maple yet! It will be very helpful in what is to come!) Then:

3. Fill in the recording strips below the graphic you produced in Task 1 with +’s and –’s using the graph of f as a guide.

4. a) Fill in the recording strips on the graphic you produced in Task 2(b) using the information you computed in Task 2(a) as a guide. (You will need to evaluate f , f' , and f'' between their respective zeroes to determine where they are positive and negative. This is where Maple can still be of use to you: using it will make your evaluations easier.)

b) Use the information you recorded in part (a) of this task to sketch the graph of f in the space provided.

Your lab report will be a hard-copy of your typed input and Maple's responses (both text and hand-labeled graphics).

Comments

To graph an arbitrary function f we need to determine not only where f and its first and second derivatives f' and f'' are positive, negative, and zero, but also where they do not exist.

The points where:

1. $f(x) = 0$ are the x -intercepts of the graph of f ,
2. $f(x)$ does not exist are the x -intercepts of the vertical asymptotes of the graph of f ,
3. $f'(x) = 0$ are critical points of the graph of f ,
4. $f'(x)$ does not exist are the x -intercepts of the vertical tangents to the graph of f ,
5. $f''(x) = 0$ and $f''(x)$ do not exist are the x -intercepts of the possible inflection points of the graph of f .

For a function f for which $f(x)$, $f'(x)$, and/or $f''(x)$ do not exist for certain values of x , we draw and label vertical bars in our recording strips with $\pm\infty$.