Newton's Method: Part 1

Michael Penna, Indiana University – Purdue University, Indianapolis

Objective

To study Newton's Method.

Narrative

Newton's Method is a method for finding an approximation to a value of x for which $f(x) = 0$ (a *zero* of f) given an initial approximation x_1 to x. This method uses the iterative equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \ldots,$$

x_1 being given, to obtain successive approximations to a zero of f.

Task

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands apply Newton's Method to finding the zero x of $f(x) = 8x^3 - 6x + 1$ given an initial estimate of $x_1 = 1.0$. The effect of each command is described in the right-hand column for your reference. Note that Maple uses brackets `[ ]` to identify subscripts.

`> # Your name, today´s date`	
`> # Newton's Method:  Part 1`	
`> restart;`	Clear Maple's memory.
`> f := x -> 8*x^3-6*x+1;`	Define f.
`> plot(f(x),x=-2..2,y=-2..4);`	Plot the graph of f.
`> g := x -> x - f(x)/D(f)(x);`	Since $g(x) = x - \frac{f(x)}{f'(x)}$, it will follow that $x_{n+1} = g(x_n)$ for each $n > 1$.
`> x[1] := 1.0;`	Let $x_1 = 1.0$. (Make sure you type 1.0, not 1: it'll make a difference!)
`> for n from 1 to 6 do x[n+1] := g(x[n]) end do;`	Iterate!

At this time make a hard-copy of your typed input and Maple's responses. Then:

2. Observe that (under ideal circumstances) more and more decimal places "stabilize" the more times Newton's Method is iterated. To 8 decimal places of accuracy, what number x_∞ does it appear that x_n is converging to?

3. Plot and label the points P_1, P_2, and P_∞ on the x-axis whose x-coordinates are x_1, x_2, and x_∞.

Your lab report will be a hard-copy of your typed input and Maple's responses (both text and hand-labeled graphics).