Newton's Method

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Objective

To study Newton's Method.

Narrative

Newton's Method is a method for finding an approximation to a value of x for which $f(x) = 0$ (or a *zero* of f) given an initial approximation x_1 to x. This method uses the iterative equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \ldots,$$

x_1 being given, to obtain successive approximations x_2, x_3, …, to x.

Task

1. a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands apply Newton's Method to finding the root of $f(x) = 8x^3 - 6x + 1 = 0$ given an initial estimate of $x_1 = 1.0$. The effect of each command is described in the right-hand column for your reference. Note that Maple uses brackets [ ] to denote subscripts.

`> # Your name, today´s date`	
`> # Newton's Method`	
`> # Task 1`	
`> restart;`	Clear Maple's memory.
`> f := x -> 8*x^3-6*x+1;`	Define f.
`> plot(f(x),x=-2..2,y=-2..4);`	Plot the graph of f.
`> g := x -> x - f(x)/D(f)(x);`	Let $g(x) = x - \dfrac{f(x)}{f'(x)}$.
`> x[1] := 1.0;`	Let $x_1 = 1.0$. (Make sure you type 1.0, not 1: it'll make a difference!)
`> for n from 1 to 6 do x[n+1] := g(x[n]) end do;`	This computes $x_{n+1} = g(x_n)$ for $n = 1, 2, \ldots$.

The idea behind the `for` loop is that since $g(x) = x - f(x)/f'(x)$, $x_{n+1} = g(x_n)$ for every $n > 1$.

At this time make a hard-copy of your typed input and Maple's responses. Then:

b) To 8 decimal places of accuracy, what number x_∞ does it appear that x_n is converging to?

c) Plot and label the points P_1, P_2, and P_∞ on the x-axis whose x-coordinates are x_1, x_2, and x_∞.

2. Use Newton's Method to estimate the solutions to the equation $\sin x = x^2 - 1$ by proceeding as follows:

a) Let $f(x) = \sin x - (x^2 - 1)$ and plot the graph of f for $x \in [-5, 5]$ and $y \in [-10, 2]$. (The graph will help you estimate the roots of $f(x) = 0$.)

b) Estimate each zero of $f(x)$ by a number x_1, and iterate Newton's Method enough times to obtain an approximation to the root to 8 decimal places of accuracy. (*Note*: Your initial estimate of each root must be close enough to the root for Newton's Method to converge: if it's not close enough, the values x_1, x_2, x_3, … might not converge to the value you're interested in!)

Again, make a hard-copy of your typed input and Maple's responses. Then:

c) To 8 decimal places of accuracy, what number x_∞ does it appear that x_n is converging to?

d) Plot and label the points P_1, P_2, and P_∞ on the x-axis whose x-coordinates are x_1, x_2, and x_∞.

Your lab report will be a hard-copy of your typed input and Maple's responses (both text and hand-labeled graphics).