

Ballistics

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Objective

To illustrate an important application of differentiation to ballistics.

Narrative

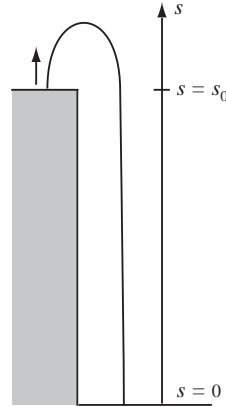
If a projectile is fired vertically upward with an initial velocity of v_0 ft/sec from an initial position s_0 ft above the ground (see the figure to the right), then (neglecting air resistance) after t sec the projectile is

$$s = s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$$

feet above the ground, where $g = 32$ ft/sec² is acceleration due to gravity, and the velocity of the projectile is

$$v = v(t) = D_t(s(t)) = -gt + v_0$$

feet per second. (If we use metric units then $g = 9.8$ m/sec².)



Tasks

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed.

```
> # Your name, today's date
```

```
> # Ballistics
```

```
> restart;
```

```
> g := 32; t0 := 0; s0 := 100; v0 := 128;
```

```
> s := t -> -0.5*g*t^2+v0*t+s0;
```

```
> v := D(s);
```

```
> t1 := 1.0; s(t1);
```

Clear Maple's memory.

Let $g = 32$, $t_0 = 0$, $s_0 = 100$, and $v_0 = 128$.

(In this project we use English units.)

Let $s = s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$.

Let $v = v(t) = D_t(s(t))$.

Let $t_1 = 1.0$, and compute $s(t_1)$.

2. By using trial-and-error, change the value of t_1 in the last line you typed until you obtain the value of t_1 greater than t_0 for which $s(t_1)$ is within 2 decimal places of 0.

3. Continue by typing the command lines in the left-hand column below into Maple in the order in which they are listed.

```
> plot(s(t),t=t0..t1);
```

```
> v(t1);
```

```
> t_smax := fsolve(v(t)=0,t);
```

```
> s(t_smax);
```

Plot s as a function of t for $t \in [t_0, t_1]$.

Find the velocity $v(t_1)$ with which the projectile strikes the ground.

Find the time t_{smax} at which the velocity of the projectile is 0 (the time at which the projectile has reached its maximum altitude).

Find the maximum altitude $s(t_{smax})$ of the projectile.

At this time make a hard-copy of your typed input and Maple's responses. Then:

4. Label the coordinate axes in the graphic you produced by hand (one should be a t -axis, and the other an s -axis), and plot and label the location of the projectile at times $t = t_0$, $t = t_1$, and $t = t_{smax}$.

Your lab report will be a hard-copy of your typed input and Maple's responses (both text and hand-labeled graphics).

Comments

In this project we found the time it takes a projectile fired vertically upward from 100 ft above the ground at an initial velocity of 128 ft/sec, to hit the ground, the velocity with which it hits the ground, the time it takes to achieve its maximum altitude, and its maximum altitude. A slightly different problem involves analyzing the motion of a projectile fired at an elevation angle θ above the horizontal, with an initial velocity of v_0 ft/sec from a point s_0 ft above the ground. A graphic illustrating such a situation can be produced with the following Maple code:

```
> restart;
> g := 32; t0 := 0; s0 := 100; v0 := 128; theta := Pi/4;
> x := t -> v0*cos(theta)*t;
> y := t -> -0.5*g*t^2+v0*sin(theta)*t+s0;
> t1 := 6.6034; evalf(y(t1));
> plot([x(t),y(t),t=t0..t1]);
```

In this graphic, however, the horizontal axis is an x -axis, the vertical axis is a y -axis, and there is no t -axis! Thus there is a basic difference between what is being illustrated by this graph, and what was illustrated by the graph you drew earlier in this project.