# Ballistics

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***Objective***

To illustrate an important application of differentiation to ballistics.

***Narrative***

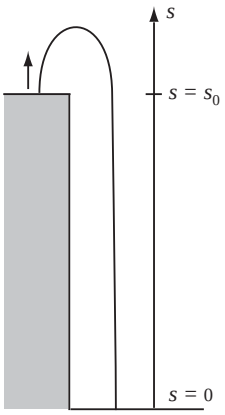
If a projectile is fired vertically upward with an initial velocity of $v_0$ m/sec from an initial position $s_0$ meters above the ground (see the figure to the right), then (neglecting air resistance) after $t$ sec the projectile is

$$s = s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$$

meters above the ground, where $g = 9.8$ m/sec$^2$ is acceleration due to gravity, and the velocity of the projectile is

$$v = v(t) = D_t(s(t)) = -gt + v_0$$

meters per second. (If we use English units then $g = 32$ ft/sec$^2$.)




***Tasks***

**1**. Type the command lines in the left-hand column below into Maple in the order in which they are listed.

| | |
|---|---|
| `> # Your name, today´s date` | |
| `> # Ballistics` | |
| `> restart;` | Clear Maple's memory. |
| `> g := 9.8; t0 := 0; s0 := 100; v0 := 128;` | Let $g = 9.8$, $t_0 = 0$, $s_0 = 100$, and $v_0 = 128$. (In this project we use metric units.) |
| `> s := t -> -0.5*g*t^2+v0*t+s0;` | Let $s = s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$. |
| `> v := D(s);` | Let $v = v(t) = D_t(s(t))$. |
| `> t1 := 1.0; s(t1);` | Let $t_1 = 1.0$, and compute $s(t_1)$. |

**2**. By using trial-and-error, change the value of `t1` in the last line you typed until you obtain the value of `t1` *greater than* `t0` for which `s(t1)` is within 2 decimal places of 0.

**3**. Continue by typing the command lines in the left-hand column below into Maple in the order in which they are listed.

| | |
|---|---|
| `> plot(s(t),t=t0..t1);` | Plot $s$ as a function of $t$ for $t \in [t_0, t_1]$. |
| `> v(t1);` | Find the velocity $v(t_1)$ with which the projectile strikes the ground. |
| `> t_smax := fsolve(v(t)=0,t);` | Find the time $t_{smax}$ at which the velocity of the projectile is 0 (the time at which the projectile has reached its maximum altitude). |
| `> s(t_smax);` | Find the maximum altitude $s(t_{smax})$ of the projectile. |

At this time make a hard-copy of your typed input and Maple's responses. Then:

**4**. Label the coordinate axes in the graphic you produced by hand (one should be a $t$-axis, and the other an $s$-axis), and plot and label the location of the projectile at times $t = t_0$, $t = t_1$, and $t = t_{smax}$.

Your lab report will be a hard-copy of your typed input and Maple's responses (both text and hand-labeled graphics).

***Comments***

In this project we found the time it takes a projectile fired vertically upward from 100 meters above the ground at an initial velocity of 128 m/sec, to hit the ground, the velocity with which it hits the ground, the time it takes to achieve its maximum altitude, and its maximum altitude. A slightly different problem involves analyzing the motion of a projectile fired at an elevation angle $\theta$ above the horizontal, with an initial velocity of $v_0$ m/sec from a point $s_0$ meters above the ground. A graphic illustrating such a situation can be produced with the following Maple code:

```
> restart;
> g := 9.8; t0 := 0; s0 := 100; v0 := 128; theta := Pi/4;
> x := t -> v0*cos(theta)*t;
> y := t -> -0.5*g*t^2+v0*sin(theta)*t+s0;
> t1 := 6.6034; evalf(y(t1));
> plot([x(t),y(t),t=t0..t1]);
```

In this graphic, however, the horizontal axis is an $x$-axis, the vertical axis is a $y$-axis, and there is no $t$-axis! Thus there is a basic difference between what is being illustrated by this graph, and what was illustrated by the graph you drew earlier in this project.