

Areas of Type I and Type II Regions

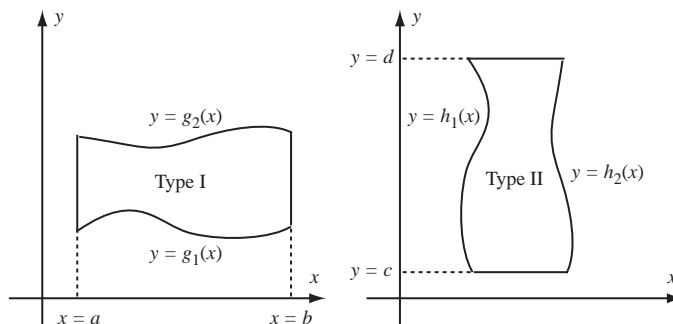
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Objective

To illustrate how to find the areas of Type I and Type II regions.

Narrative

There are two basic types of regions in the xy -coordinate plane: Type I regions and Type II regions. Type I regions are regions bounded by lines $x = a$ and $x = b$, where $a < b$, and curves $y = g_1(x)$ and $y = g_2(x)$, where $g_1(x) < g_2(x)$ for $a < x < b$. Type II regions are bounded by lines $y = c$ and $y = d$, where $c < d$, and curves $x = h_1(y)$ and $x = h_2(y)$, where $h_1(y) < h_2(y)$ for $c < y < d$. Every region in the xy -coordinate plane can be subdivided into regions of Type I and Type II, and hence to find the area of any region in the plane it suffices to know how to find the area of regions of Type I and Type II.



The area of the Type I and Type II regions described above are, respectively,

$$\int_{x=a}^b (g_2(x) - g_1(x)) dx \quad \text{and} \quad \int_{y=c}^d (h_2(y) - h_1(y)) dy.$$

Task

1. a) Type the command lines below into Maple in the order in which they are listed. They produce a graphic of the Type I region R bounded on the left by $x = 1$, on the right by $x = 3$, below by $y = 1/2 - x/6$, and above by $y = \sqrt{x}$, and find the area of R .

```
> # Your name, today's date
> # Areas of Type I and Type II Regions
> restart;
> with(plots):
> # Task 1a
> implicitplot({x=1,x=3,y=1/2-x/6,y=sqrt(x)},x=0..4,y=0..2);
> evalf(int(sqrt(x)-(1/2-x/6),x=1..3));
```

b) Repeat part (a) for the Type 1 region R bounded on the left by $x = -1$, on the right by $x = 2$, below by $y = x^2$, and above by $y = x + 3$.

2. a) Continue by typing the command lines below into Maple in the order in which they are listed. They produce a graphic of the Type II region R bounded on the bottom by $y = -2$, on the top by $y = 1$, on the left by $x = y - 1$, and on the right by $x = y^2$, and find the area of R .

```
> # Task 2a
> implicitplot({y=-2,y=1,x=y-1,x=y^2},x=-4..4,y=-3..2);
> evalf(int(y^2-(y-1),y=-2..1));
```

b) Repeat part (a) for the Type 2 region R bounded on the bottom by $y = -1$, on the top by $y = 2$, on the left by $x = y + 3$, and on the right by $x + 2 = y^2$.

At this time make a hard-copy of your typed input and Maple's responses. Then:

3. On each of the graphics you produced for Tasks 1 and 2, by hand:

- a) label each side of the region R with its equation, and
- b) shade in the region R .

Your lab report will consist of the figures you drew by hand, your typed input and Maple's responses (both text and hand-labeled graphics).