Inverse Functions

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Objective

To investigate inverse functions.

Narrative

In this project we discuss inverse functions. Some of the key things you should know about inverse functions are:

1. To show that a function f has an inverse, you need to show that f is $1-1$. You can do this by showing that either: a) $f'(x) > 0$ for all x in the domain of f, or b) $f'(x) < 0$ for all x in the domain of f.
2. You can find the inverse f^{-1} of a simple function f by solving the equation $y = f(x)$ for x in terms of y; the resulting equation is $x = f^{-1}(y)$. (So if you are looking for $f^{-1}(x)$, simply interchange x and y in this equation.) Remember that this only works for simple functions f.
3. One way to check your work in computing the inverse f^{-1} of a simple function f is to verify that: a) $f^{-1}(f(x)) = x$ for all x in the domain of f, and b) $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}.
4. The graph of the inverse f^{-1} of a function f is the reflection of the graph of f in the line $y = x$.
5. The derivative of f is related to the derivative of f^{-1} by the equation $D_x(f^{-1}(x)) = 1/D_y(f(y))$.

Tasks

1. Type the command lines below into Maple in the order in which they are listed. The effect of these commands is to graph $f(x) = \sin x$, $g(x) = f^{-1}(x) = \arcsin x$, and $y = x$ on one set of coordinate axes.

```
> # Your name, today´s date
> # Inverse Functions
> restart;
> f := x -> sin(x);
> g := x -> arcsin(x);
> plot({f(x),g(x),x},x=0..Pi/2,scaling=unconstrained);
```

At this point, make a hard-copy of your typed input and Maple's responses (both text and graphics). Then:

2. Label the graphs of f and $g = f^{-1}$.

3. Plot and label the points $P_0(1, \sin 1)$ and $Q_0(\sin 1, 1)$.

4. Using a straightedge, very carefully draw the tangent line to the graph of f at P_0 and the tangent line to the graph of $g = f^{-1}$ at Q_0.

5. a) Plot a point P_1 other than P_0 on the tangent line to the graph of f at P_0. By eye, estimate the coordinates of P_1, and label P_1 with its coordinates.

b) Plot a point Q_1 other than Q_0 on the tangent line to the graph of $g = f^{-1}$ at Q_0. By eye, estimate the coordinates of Q_1, and label Q_1 with its coordinates.

c) Use P_0 and P_1 to estimate the slope of the tangent line to the graph of f at P_0, and label this tangent line with its slope.

d) Use Q_0 and Q_1 to estimate the slope of the tangent line to the graph of $g = f^{-1}$ at Q_0, and label this tangent line with its slope.

6. a) What should the relationship between the two slopes you computed in Task 5 be?

b) Why should this relationship be true?

c) Numerically verify the relationship between the two slopes you computed in Task 5.

Your lab report will be a hard-copy of your typed input Maple's responses (both text and hand-drawn graphics), and your written responses.