Inverse Functions

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Objective

To investigate inverse functions.

Narrative

In this project we discuss inverse functions. Some of the key things you should know about inverse functions are:

1. To show that a function f has an inverse, you need to show that f is $1-1$. You can do this by showing that either: a) $f'(x) > 0$ for all x in the domain of f, or b) $f'(x) < 0$ for all x in the domain of f.

2. You can find the inverse f^{-1} of a simple function f by solving the equation $y = f(x)$ for x in terms of y; the resulting equation is $x = f^{-1}(y)$. (So if you are looking for $f^{-1}(x)$, simply interchange x and y in this equation.) Remember that this only works for simple functions f.

3. One way to check your work in computing the inverse f^{-1} of a simple function f is to verify that: a) $f^{-1}(f(x)) = x$ for all x in the domain of f, and b) $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}.

4. The graph of the inverse f^{-1} of a function f is the reflection of the graph of f in the line $y = x$.

5. The derivative of f is related to the derivative of f^{-1} by the equation $D_x(f^{-1}(x)) = 1/D_y(f(y))$.

Tasks

1. Type the command lines below into Maple in the order in which they are listed. The effect of these commands is to graph $f(x) = \sin x$ and $g(x) = f^{-1}(x) = \arcsin x$, as well as the tangent line to f at $P_1(1, \sin 1)$ and the tangent to g at $Q_1(\sin 1, 1)$.

```
> # Your name, today´s date
> # Inverse Functions
> restart;
> with(plots):
> with(Student):
> f := x -> sin(x);
> g := x -> arcsin(x);
> plot0 := showtangent(f(x),x=1,x=0..Pi/2):
> plot1 := showtangent(g(x),x=sin(1),x=0..1):
> display({plot0,plot1},scaling=constrained);
```

2. Label the graphs of f and $g = f^{-1}$.

3. Plot and label the points $P_0(1, \sin 1)$ and $Q_0(\sin 1, 1)$.

4. a) Plot a point P_1 other than P_0 on the tangent line to the graph of f at P_0. By eye, estimate the coordinates of P_1, and label P_1 with its coordinates.

b) Plot a point Q_1 other than Q_0 on the tangent line to the graph of $g = f^{-1}$ at Q_0. By eye, estimate the coordinates of Q_1, and label Q_1 with its coordinates.

c) Use P_0 and P_1 to estimate the slope of the tangent line to the graph of f at P_0, and label this tangent line with its slope.

d) Use Q_0 and Q_1 to estimate the slope of the tangent line to the graph of $g = f^{-1}$ at Q_0, and label this tangent line with its slope.

6. a) What should the relationship between the two slopes you computed in Task 4 be?

b) Why should this relationship be true?

c) Numerically verify the relationship between the two slopes you computed in Task 4.

Your lab report will be a hard-copy of your typed input Maple's responses (both text and hand-drawn graphics), and your written responses.