

Partial Fractions

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Objective

To illustrate how Maple can be used to integrate a rational function by partial fractions.

Narrative

One of the challenging aspects of integration by partial fractions is the algebra involved in obtaining the partial fraction decomposition of the integrand. In this project we illustrate how Maple can be used to find

$$\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx.$$

Our reason for doing this is two-fold: First, it might help you with your homework. Second, it is important to learn how to do a partial fraction decomposition of a rational function since doing so is a basic skill required in solving differential equations using Laplace transforms.

In this project we use the command `coeff`: “`coeff(p(x), x, n)`” is the coefficient of x^n in the polynomial expression $p(x)$.

Task

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands are aimed at evaluating

$$\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx.$$

<pre>> # Your name, today's date > # Partial Fractions > # Task 1 > restart; > num := x^4-2*x^2+4*x+1; den := x^3-x^2-x+1;</pre>	Let <code>num</code> and <code>den</code> denote the numerator and denominator of the fraction.
<pre>> quo(num,den,x); rem(num,den,x); > factor(den);</pre>	Find the quotient and remainder of <code>num/den</code> . Now factor the denominator.
<pre>> 4*x/((x+1)*(x-1)^2) = A/(x-1) + B/(x-1)^2 + C/(x+1);</pre>	Thus our partial fraction decomposition will look like this ...
<pre>> (x+1)*(x-1)^2*%; > eq := simplify(%); > eq0 := coeff(lhs(eq),x,0) = coeff(rhs(eq),x,0);</pre>	To find <code>A</code> , <code>B</code> , and <code>C</code> , first clear denominators and simplify.
<pre>> eq1 := coeff(lhs(eq),x,1) = coeff(rhs(eq),x,1); > eq2 := coeff(lhs(eq),x,2) = coeff(rhs(eq),x,2);</pre>	Now equate the constant on the left-hand side of <code>eq</code> to that on the right-hand side, the coefficient of x on the left to the coefficient of x on the right, and the coefficient of x^2 on the left to the coefficient of x^2 on the right.
<pre>> solve({eq0,eq1,eq2},{A,B,C}); > simplify(1/(x-1)+2/(x-1)^2-1/(x+1)); > int(x+1+1/(x-1)+2/(x-1)^2-1/(x+1),x); > simplify(diff(%,x));</pre>	Now solve <code>eqs 0, 1, and 2</code> for <code>A</code> , <code>B</code> , and <code>C</code> . Let's check our work. Finally let's compute the integral ... and check our answer by differentiating.

2. Use Maple as illustrated above, to evaluate

$$\int \frac{x^4 + x^3 + 2x + 1}{x^3 - x^2} dx.$$

At this time make a hard-copy of your typed input and Maple's responses. This hard-copy will be your lab report.

Comments

1. The purpose of this project was to illustrate how Maple can be used to help you check the computations that you would otherwise do by hand in completing your homework. If you just want to check whether your answer is correct, you can either:
 - a) see what you get when you ask Maple to add the partial fractions, or
 - b) use the Maple command `convert(f(x), parfrac, x)` to compute the partial fraction decomposition of the expression $f(x)$ in the variable x ; see Maple's Help for further details.
2. In this project we admittedly limited our attention to integrating two rational functions that have some fairly simple partial fraction decompositions. More complicated functions can easily require further manipulation; again see Maple's Help for further commands and details on using them.