

# Approximate Integration

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## Objective

To illustrate and compare Riemann sums, the Trapezoidal Rule, and Simpson's Rule.

## Narrative

In this project we illustrate and compare Riemann sums, the Trapezoidal Rule, and Simpson's Rule by looking at how they approximate  $\int_{x=1}^2 \frac{dx}{x} = \ln 2$ .

In this project we use the command `type`: “`type(n,odd)`” is true if  $n$  is odd and false if  $n$  is even, and “`type(n,even)`” is true if  $n$  is even and false if  $n$  is odd. We also use the `if/then/elif/else/end if` control structure.

## Task

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. They provide Maple some initial information.

> # Your name, today's date	
> # Approximate Integration	
> restart;	Clear Maple's memory.
> f := x -> 1/x;	Let $f(x) = 1/x$ .
> NInts := 8;	Let the number of intervals $NInts = 8$ .
> a := 1.0; b := 2.0; dx := (b-a)/NInts;	Let $a = 1, b = 2$ , and $dx = (b-a)/NInts$ .
> Actual := int(f(x), x=a..b);	What is $\int_{x=a}^b f(x) dx$ ?

a) Continue by typing the command lines in the left-hand column below into Maple in the order in which they are listed. These commands approximate our integral by Riemann sums.

> # Task 1a: Riemann Sums	
> Riemann := 0.0;	Initialize Riemann to 0.
> for n from 1 to NInts do	
Riemann := Riemann + f(a+n*dx)*dx	Compute the Riemann sum.
end do;	
> plot(f(x), x=a..b);	Draw the graph of $f$ .
> abs(Actual-Riemann);	The Riemann sum error.

b) Continue by typing the command lines in the left-hand column below into Maple in the order in which they are listed. These commands approximate our integral by the Trapezoidal Rule.

> # Task 1b: The Trapezoidal Rule	
> wt := n -> if n=0 then 1	Define the weighting function $wt$ .
elif n=NInts then 1	
else 2 end if;	
> for n from 0 to NInts do wt(n) end do;	Check the weighting function.
> Trap := 0.0;	Initialize Trap to 0.
> for n from 0 to NInts do	
Trap := Trap + wt(n)*f(a+n*dx)*dx/2	Apply the Trapezoidal Rule.
end do;	
> plot(f(x), x=a..b);	Draw the graph of $f$ .
> abs(Actual-Trap);	The Trapezoidal Rule error.

c) Continue by typing the command lines in the left-hand column below into Maple in the order in which they are listed. These commands approximate our integral by Simpson's Rule.

> # Task 1c: Simpson's Rule	
> wt := n -> if n=0 then 1	Define the weighting function <i>wt</i> .
elif n=NInts then 1	
elif type(n,odd) then 4	
elif type(n,even) then 2 end if;	
> for n from 0 to NInts do wt(n) end do;	Check the weighting function.
> Simpson := 0.0;	Initialize Simpson to 0.
> for n from 0 to NInts do	
Simpson := Simpson + wt(n)*f(a+n*dx)*dx/3	Apply Simpson's Rule.
end do;	
> plot(f(x),x=a..b);	Draw the graph of <i>f</i> .
> abs(Actual-Simpson);	Simpson's Rule error.

At this time, make a hard-copy of your typed input and Maple's responses. Then:

2. On each of the graphs you created in Tasks 1(a), (b), and (c), sketch and shade in the region whose area approximates  $\int_{x=1}^2 \frac{dx}{x}$  by Riemann sums, the Trapezoidal Rule, and Simpson's Rule, respectively, when  $n = 8$ .

3. When  $n = 8$ , which method — Riemann sums, the Trapezoidal Rule, or Simpson's Rule — provides the best estimate of  $\int_{x=1}^2 \frac{dx}{x} = \ln 2$ ? Which provides the worst?

Your lab report will be a hard-copy of your typed input and Maple's responses (both text and hand-drawn graphics).

### Comments

In practice, most definite integrals cannot be evaluated in closed form, and techniques such as the Trapezoidal Rule and Simpson's Rule must be applied to find numerical approximations. Maple actually has built in commands that make finding such approximations quite easy. We did not use them in this project, however, since our goal was to illustrate how such methods work, not just how to use certain commands.