Variation and Proportion

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Objective

In this project we discuss variation and proportion. This project involves computations you can make by hand.

Narrative

The terms "variation" and "proportion" are an important part of the language of science and engineering.

A quantity Q is said to *vary directly* as a quantity x, written $Q \propto x$, if $Q = kx$ for some constant k (known as the *constant of proportionality*). (The phrases *vary with* and *is proportional to* are often used interchangeably with *vary directly as*.) That is,

$$Q \propto x \quad \text{if and only if} \quad Q = kx$$

for some constant k.

Example 1: Experiments have shown that the force F required to stretch a spring a distance x units out of equilibrium varies directly as x, as long as x is small. That is,

$$F \propto x \quad \text{or} \quad F = kx$$

for some constant k (known as the *spring constant* of the spring). This is known as *Hooke's Law*. The size of k depends on the physical characteristics of the spring: If a force of 6 pounds is required to stretch a spring 2 inches out of equilibrium, then $6 = 2k$ so $k = 3$ pounds per inch. It would take a force of $F = 3 * 1 = 3$ pounds to stretch this spring 1 inch out of equilibrium, and a force of $F = 3 * 3 = 9$ pounds to stretch it 3 inches out of equilibrium. If a force of 4 pounds were required to stretch a different spring 2 inches out of equilibrium, then $4 = 2k$ so $k = 2$ pounds per inch. It would take a force of $F = 2 * 1 = 2$ pounds to stretch this spring 1 inch out of equilibrium, and a force of $F = 2 * 3 = 6$ pounds to stretch it 3 inches out of equilibrium.

The quantity x we referred to in our description of variation can be *any* type of algebraic expression.

Example 2: The area A of a circle varies as the square r^2 of the radius r of the circle, or $A \propto r^2$ since $A = \pi r^2$. (In this case, the constant k of proportionality is π.)

A quantity Q is said to *vary inversely* as a quantity x, that is

$$Q \propto \frac{1}{x} \quad \text{if and only if} \quad Q = \frac{k}{x}$$

for some constant k.

Example 3: If the temperature of some quantity of gas that has been confined to a cylinder is kept constant, then the pressure P of the gas varies inversely as the volume V of the gas; that is

$$P \propto \frac{1}{V} \quad \text{or} \quad P = \frac{k}{V}$$

for some constant k. Again, the size of k depends on the temperature and chemical properties of the gas.

More than one form of variation can arise in an application; in this case we say that one quantity Q *varies jointly* with several other quantities.

Example 4: The *Universal Law of Gravitation* states that the gravitational force F between two bodies of masses m_1 and m_2 varies jointly as the product of their masses and inversely as the square of the distance d between the centers of mass of the objects; that is,

$$F \propto \frac{m_1 m_2}{d^2} \quad \text{or} \quad F = G\frac{m_1 m_2}{d^2}$$

for a constant G (known as the *universal gravitational constant*).

Exercises

1. If y varies directly as x, and $y = 3$ when $x = 4$, find the constant k of proportionality, and the value of y when $x = 6$.

2. If y varies directly as x, and $y = 5$ when $x = 2$, find the constant k of proportionality, and the value of y when $x = 3$.

3. If y varies inversely as x, and $y = 3$ when $x = 4$, find the constant k of proportionality, and the value of y when $x = 6$.

4. If y varies inversely as x, and $y = 5$ when $x = 2$, find the constant k of proportionality, and the value of y when $x = 3$.

5. If y varies as the square of x, and $y = 3$ when $x = 4$, find the constant k of proportionality, and the value of y when $x = 6$.

6. If y varies as the square of x, and $y = 3$ when $x = 2$, find the constant k of proportionality, and the value of y when $x = 6$.

7. If y varies inversely as the square of x, and $y = 3$ when $x = 4$, find the constant k of proportionality, and the value of y when $x = 6$.

8. If y varies inversely as the square of x, and $y = 2$ when $x = 3$, find the constant k of proportionality, and the value of y when $x = 3$.

9. If z varies directly as the square of x and inversely as y, and $z = 3$ when $x = 4$ and $y = 5$, find the constant k of proportionality, and the value of z when $x = 2$ and $y = 6$.

10. If z varies directly as the square of x and inversely as y, and $z = 3$ when $x = 4$ and $y = 5$, find the constant k of proportionality, and the value of z when $x = 4$ and $y = 1$.