

Force and Work

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Objective

In this project we discuss force and work in anticipation of an important application of Calculus. This project involves computations you should be able to make by hand.

Force

A force, intuitively, is some sort of push or pull. One of the most familiar forces is the force of gravity: this is the force exerted by the earth that pulls us down. Forces have both direction and magnitude, and forces can be measured with a spring scale. The direction of gravity, for example, is down, towards the center of the earth, and the magnitude of gravity on an individual is the individual's weight. An individual's weight can be measured by having them stand on a floor scale or sit on the seat of a hanging scale. The British unit of force is the pound (lb), and the SI unit of force is the Newton (N).

Let us take a closer look at springs. Experiments have shown that the force F required to stretch a spring a distance x out of equilibrium varies directly as x , as long as x is small (see Fig. 1). That is,

$$F \propto x, \quad \text{or} \quad F = kx$$

for some constant k known as the *spring constant* of the spring. This is known as *Hooke's Law*. The size of k depends on the physical characteristics of the spring: If a force of 6 pounds is required to stretch a spring 2 inches out of equilibrium, then $6 = 2k$ so $k = 3$ pounds per inch. Consequently $F = 3x$, and it would take a force of $F = 3 * 1 = 3$ pounds to stretch this spring 1 inch out of equilibrium, and a force of $F = 3 * 3 = 9$ pounds to stretch it 3 inches out of equilibrium. If a force of 4 pounds were required to stretch a different spring 2 inches out of equilibrium, then $4 = 2k$ so $k = 2$ pounds per inch. Consequently $F = 2x$, and it would take a force of $F = 2 * 1 = 2$ pounds to stretch this spring 1 inch out of equilibrium, and a force of $F = 2 * 3 = 6$ pounds to stretch it 3 inches out of equilibrium.

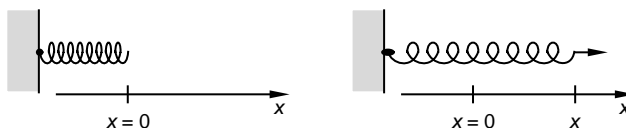


Figure 1: A Spring

Many forces act simultaneously on most bodies. For example, a car being driven down the road is subject not only to the force of gravity, but also the force (that moves the car forward) exerted by the car's engine, the force of air resistance, and the force of friction where the tires meet the road. If an object is at rest, then the sum of all forces acting on the object is zero. In the case of a spring stretched out of equilibrium by an object attached to the spring, for example, two equal but opposite forces act on the object when it is at rest: one force is the upward force of the spring, and the other force is the downward force of gravity.

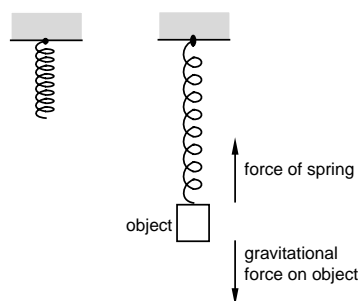


Figure 2: An Object at Rest

Let us now return to our discussion of gravity and weight. Newton's Second Law of Motion states that the magnitude of the acceleration of an object is proportional to the net force acting on it and inversely proportional to its mass

$$a \propto \frac{F}{m}$$

and that the direction of acceleration is the same as the direction of the net force. Assuming units are chosen so that the constant of proportionality is 1, it follows that

$$F = ma.$$

Now Galileo observed that all objects dropped near the surface of the earth fall with the same acceleration g , if air resistance is neglected. Combining Galileo's observation with Newton's Second Law of Motion, it follows that the magnitude of the force due to gravity on an object near the surface of the earth — the object's weight —

$$w = F = mg.$$

The British units for weight and mass are pounds and slugs, respectively, and $g = 32 \text{ ft/sec}^2$; since the British units for acceleration are ft/sec^2 , it follows that the units for pounds are slug ft/sec^2 . The SI units for weight and mass are Newtons and kilograms, respectively, and $g = 4.9 \text{ m/sec}^2$; since the SI units for acceleration are m/sec^2 , it follows that the units for Newtons are kg m/sec^2 .

Just as your weight is evidence that one force of the earth — namely the force of gravity — is pulling you down, the fact that you don't fall towards the center of the earth (as you stand on it) is evidence that there is a second force exerted by the earth that is pushing you up. This second force is the structural force of the surface of the earth. The magnitude of this force is the same as your weight, and as you stand on the earth you are in much the same state of equilibrium as an object at the end of the spring when the object is at rest.

Exercises

- What is the spring constant of a spring which is:
 - stretched 3 in. from its natural length by a weight of 2 lb?
 - stretched 5 in. from its natural length by a weight of 3 lb?
 - stretched 3 cm from its natural length by a weight of 0.5 N?
 - stretched 4 cm from its natural length by a weight of 0.75 N?
 - What is the spring constant of a spring which is:
 - 3 in. long if a weight of 2 lbs stretches it to a length of 4 in?
 - 4 in. long if a weight of 3 lbs stretches it to a length of 6 in?
 - 5 cm long if a weight of 0.1 lbs stretches it to a length of 7 cm?
 - 4 cm long if a weight of 0.3 lbs stretches it to a length of 9 cm?
 - If a force of:
 - 2 lb stretches a spring 4 in. from its natural length, what force stretches it 6 in. from its natural length?
 - 3 lb stretches a spring 5 in. from its natural length, what force stretches it 8 in. from its natural length?
 - 0.25 N stretches a spring 3 cm from its natural length, what force stretches it 5 cm from its natural length?
 - 0.3 N stretches a spring 4 cm from its natural length, what force stretches it 5 cm from its natural length?
 - Assuming that $g = 32 \text{ ft/sec}^2$ or 9.8 m/sec^2 , how much does an object of mass:
 - 2 slugs weigh? (b) 3 slugs weigh? (c) 2 kg weigh? (d) 5 kg weigh?
 - Assuming that $g = 32 \text{ ft/sec}^2$ or 9.8 m/sec^2 , what is the mass of an object that weighs:
 - 100 lbs? (b) 190 lbs? (c) 10 N? (d) 16 N?
 - The density of water is 1000 kg/m^3 , and the weight of water is 62.5 lb/ft^3 . (*Note:* The density of any homogeneous substance such as water is mass per unit volume.)
 - What is the density of water in slugs/ft^3 ?
 - What is the weight of water in Newtons/m^3 ?
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Work

The *work* done by a force that is constant in both magnitude and direction is defined to be the product of the magnitude of the force and the distance — in the direction of the force — through which the force acts

$$W = Fd.$$

If, for example, a force of 3 pounds is applied to lift a book 4 ft off the floor then $3 \times 4 = 12$ foot-pounds (ft-lbs) of work have been done. As this example illustrates, the British unit of work is the foot-pound: the work required to move 1 pound through a distance of 1 foot. If a force of 2 Newtons is applied to lift a book 1.5 m off the floor then $2 \times 1.5 = 3$ Newton-meters, or joules (J), of work have been done. As this example illustrates, the SI unit of work is the Newton-meter, or joule: this is the work required to exert a force of 1 Newton through a distance of 1 meter.

Three features of the definition of work deserve comment.

First, according to the definition of work, motion must take place for work to be done; in other words, a force can be applied without work being performed. If, for example, we were to lift up on the bumper of a car then we would exert a force on the car but no work would be done (at least by most of us) since the car would not move. Movement is necessary for work to be done. This may or may not agree with your intuition about what “work” should mean, but it is the way work is defined scientifically.

Second, the definition of work applies only when the magnitude and direction of a force are constant. To compute the work done by a force that *varies* in magnitude and/or direction we need the tools of calculus. An example of a force that varies in magnitude is the force required to stretch a spring out of equilibrium, or the force of gravity that acts on a bucket, initially full of water, but with a hole in the bottom through which water leaks as the bucket is lifted. The work done by a force such as this is defined by an integral. An example of a force that varies in direction is the force required to move a particle through a force field while maintaining the particle’s position along a given path. The work done by a force such as this is again defined by an integral.

Third, the definition of work assumes that the direction of the applied force is the same as the direction of the resulting motion. An example of a force whose direction is not the same as the direction of the resulting motion is a force applied to a heavy object as illustrated in Fig. 3: although the object may not be lifted off the ground, it may still be moved forward. The work done by a force such as this is the horizontal component of the force times the distance through which the force acts. We will discuss examples such as this further in our discussion of vectors.

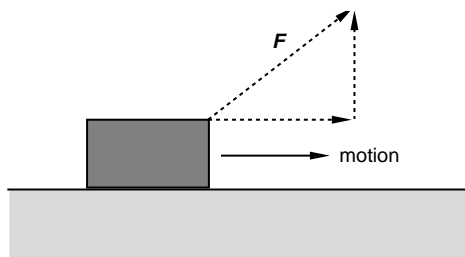


Figure 3: Applying a Force to a Heavy Object

Exercises

7. How much work is done if a force of:
- (a) 2 lbs is exerted through a distance of 3 ft?
 - (b) 3 lbs is exerted through a distance of 5 ft?
 - (c) 3 N is exerted through a distance of 2 m?
 - (d) 1.5 N is exerted through a distance of 4 m?

8. How much work is required to lift:
- (a) 2 ft^3 of water a distance of 5 ft?
 - (b) 1.5 ft^3 of water a distance of 4 ft?
 - (c) 0.5 m^3 of water a distance of 3 m?
 - (d) 0.7 m^3 of water a distance of 2 m?
9. Neglecting the weight of the bucket, how much work is required to lift a cylindrical bucket:
- (a) 1 ft in diameter containing water 8 in. deep a distance of 5 ft?
 - (b) 18 in. in diameter containing water 6 in. deep a distance of 4 ft?
 - (c) 0.3 m in diameter containing water 0.25 m deep a distance of 3 m?
 - (d) 20 cm in diameter containing water 30 cm deep a distance of 4 m?
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References

- [1] *General Physics*, Douglas C. Giancoli, Prentice-Hall, Inc., 1984.