Amplitude, Period, and Phase Shift

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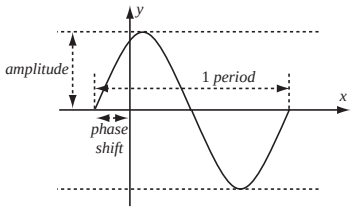
Objective

To discuss the amplitude, period, and phase shift of the sine function.

Narrative

In this project we discuss the amplitude, period, and phase shift of the sine function: If $f(x) = A\sin(Bx + C)$ then A is the amplitude, $2\pi/|B|$ is the period (the length of the domain it takes f to complete one full cycle), and, assuming $B > 0$, $-C/B$ is the phase shift.



Task

1. Type the command lines below into Mathematica in the order in which they are listed. They produce a graph of $f(x) = \sin x$ over the interval $[-2\pi, 2\pi]$.

```
In[1]:= (* Your name, today´s date *)
In[2]:= (* Amplitude, Period, and Phase Shift *)
In[3]:= Plot[Sin[x], {x,-2Pi,2Pi}]
```
Graph $\sin x$ over the interval $[-2\pi, 2\pi]$.

2. Continue by typing the following command lines into Mathematica. The effect of each command is described in the right-hand column for your reference.

`In[4]:= 3Plot[Sin[2x], {x,-2Pi,2Pi}]`	Triple the amplitude and halve the period.
`In[5]:= 3Plot[Sin[x/2], {x,-2Pi,2Pi}]`	Triple the amplitude and double the period.
`In[6]:= Plot[Sin[x+Pi/3], {x,-2Pi,2Pi}]`	Shift backward.
`In[7]:= Plot[Sin[x-Pi/3], {x,-2Pi,2Pi}]`	Shift forward.
`In[8]:= Plot[Sin[2x+Pi/3], {x,-2Pi,2Pi}]`	Halve the period and shift backward.
`In[9]:= Plot[Sin[2x-Pi/3], {x,-2Pi,2Pi}]`	Halve the period and shift forward.
`In[10]:= Plot[Sin[x/2+Pi/3], {x,-2Pi,2Pi}]`	Double the period and shift backward.
`In[11]:= Plot[Sin[x/2-Pi/3], {x,-2Pi,2Pi}]`	Double the period and shift forward.

3. Continue by typing the following command line into Mathematica. It again produces a graph of $f(x) = \sin x$ over the interval $[-2\pi, 2\pi]$.

```
In[12]:= Plot[Sin[x], {x,-2Pi,2Pi}]
```

At this point, make a hard-copy of your typed input and Mathematica's responses. Then:

4. Label the curves in each of the plots you produced in Tasks 1 and 2. For example, label the graph of $f(x) = \sin x$ in Task 1 by "$f(x) = \sin x$".

5. Next to each curve you drew in Task 2:

a) write the amplitude, period, and phase shift, and

b) plot and label the point P for which the argument of sin is 0. (For example, in the case of $\sin(x + \pi/3)$, $P = P(-\pi/3, 0)$ since $x + \pi/3 = 0$ when $x = -\pi/3$. So in this case you need to plot and label the point $P(-\pi/3, 0)$.)

6. On the graphic you created in Task 3, sketch (by hand) the graph of $f(x) = 2\sin(3x - \pi/2)$.

Your lab report will be a hard copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).

Comments

1. The frequency of $f(x) = A\sin(Bx+C)$ is the number B of complete cycles of f over the interval $[0, 2\pi]$. Frequency is related to period by

$$\text{period} = \frac{2\pi}{\text{frequency}} \quad \text{or} \quad \text{frequency} = \frac{2\pi}{\text{period}}.$$

2. If you forget the formulas for period and phase shift, you can recover them by remembering that $\sin x$ goes through one full cycle as x goes from 0 to 2π. Thus, assuming that $B > 0$, $f(x) = A\sin(Bx + C)$ goes through one full cycle as $Bx + C$ goes from 0 to 2π,

$$\text{or as} \quad Bx + C : 0 \to 2\pi \quad \text{or as} \quad Bx : -C \to 2\pi - C \quad \text{or as} \quad x : -\frac{C}{B} \to \frac{2\pi}{B} - \frac{C}{B}.$$

So the first full cycle of f begins when $x = -\dfrac{C}{B}$ (this is the phase shift) and it ends $\dfrac{2\pi}{B}$ units later (this is the length of one period) when $x = \dfrac{2\pi}{B} - \dfrac{C}{B}$.