Predicting the Slope of a Tangent Line

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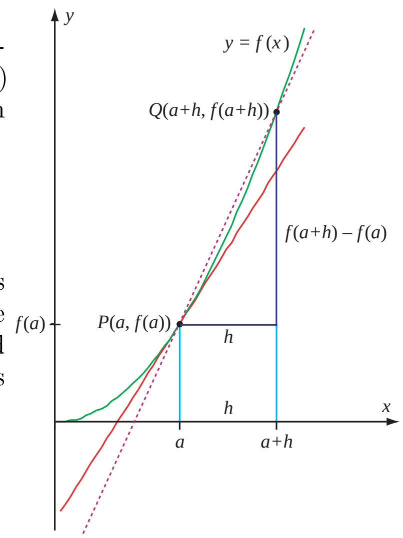
Objective

To predict the slope of a tangent line to a curve at a point.

Narrative



Recall that the tangent problem is the problem of finding the equation of the tangent line to the graph of a function f at a point $P(a, f(a))$ on the graph of f. Since the equation of any (non-vertical) line through P has an equation of the form

$$\frac{y - f(a)}{x - a} = m$$

where m is the slope of the line, solving the tangent problem reduces to finding the slope m of the tangent line to the graph of f at P. The slope m is the limit of the slopes of the secant lines through P and another point $Q(a+h, f(a+h))$ on the graph of f as $a+h$ approaches a (or as h approaches 0):

$$m = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h},$$

In this project we investigate the estimation of m from numerical data.

Tasks

1. a) Type the command lines in the left-hand column below into Mathematica in the order in which they are listed. These commands will allow you to predict the slope m of the tangent line to the graph of $f(x) = x^2/2$ at $x = a = 1$ by computing the slope $m = (f(a+h) - f(a))/h$ of the secant line that passes through $P(a, f(a))$ and $Q(a+h, f(a+h))$ for various values of h that get closer and closer to 0.

`In[1]:=` **(* Your name, today´s date *)**	
`In[2]:=` **(* Predicting the Slope of a Tangent Line *)**	
`In[3]:=` **(* Task 1a *)**	
`In[4]:=` **f[x_] := x^2/2**	Let $f(x) = x^2/2$.
`In[5]:=` **{a=1, h=0}**	Let $a = 1$ and $h = 0$.
`In[6]:=` **m=(f[a+h]-f[a])/h**	Try to compute the slope m of the "secant" line when $h = 0$. (It doesn't work, does it?!)
`In[7]:=` **h=1.6**	OK, ... let $h = 1.6$.
`In[8]:=` **For[n=0, n<4, n++,**	This is the start of a "for loop".
{h=h/2,	Decrease h by a factor of 1/2.
m=(f[a+h]-f[a])/h,	Compute the slope m of the secant line through $P(a, f(a))$ and $Q(a+h, f(a+h))$.
Print["m = ", m, 6]	Print m.
Plot[{f[x], m*(x-a)+f[a]}, {x,a-1,a+1}, AspectRatio -> Automatic]}]	Draw the graph of f and the secant line through $P(a, f(a))$ and $Q(a+h, f(a+h))$.

In the last line we used the fact that $y = f(a) + m(x - a)$ is the equation of the line whose slope is m and which passes through $P(a, f(a))$.

Observe that the secant line gets closer and closer to the tangent line as h decreases. Also observe that when $h = 0.1$, the graph of f is virtually indistinguishable from the secant / tangent line.

b) In part (a) of this task we considered the case in which h approached 0 from the right. Continue by typing the following command lines into Mathematica in the order in which they are listed. These commands provide information about what happens as h approaches 0 from the left.

`In[9]:= (* Task 1b *)`	
`In[10]:= h=1.6`	Reset h to 1.6
`In[11]:= For[n=0, n<4, n++,`	This is the start of another "for loop".
`{h=h/2,`	Decrease h by a factor of 1/2.
`m=(f[a]-f[a-h])/h,`	Compute the slope m of the secant line through $P(a, f(a))$ and $Q(a + h, f(a + h))$.
`Print["m = ", m, 6]`	Print m.
`Plot[{f[x], m*(x-a)+f[a]}, {x,a-1,a+1},`	
`AspectRatio -> Automatic]}]`	Draw the graph of f and the secant line through $P(a, f(a))$ and $Q(a + h, f(a + h))$.

At this time make a hard-copy of your typed input and Mathematica's responses. Then:

2. On each graphic you produced in Task 1, plot and label the points P and Q.

3. To 2 decimal places of accuracy, guess the slope of the tangent line to the graph of $f(x) = x^2/2$ at the origin. Justify your answer.

Your lab report will be a hard copy of your typed input and Mathematica's responses (both text and hand-labeled graphics), and your written responses.