

Differentiation

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Objective

To investigate differentiation.

Narrative

Recall that, as long as the limit exists, the first derivative f' of the function f is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

In this project we use the definition of derivative to compute f' for $f(x) = 1/(x^2 + 4)$. We also graph f and several of its derivatives to emphasize the connection between a function and its derivative: for each x in the domain of the derivative f' of f , $f'(x)$ is the slope of the tangent line to the graph of f at x ; it follows that f is increasing at x if and only if $f'(x) > 0$, and f is decreasing at x if and only if $f'(x) < 0$.

There are several different ways to write the derivative of a function in Mathematica. In this project we use the notation f' to denote the derivative of the function f . We could also write the derivative of f as $\mathbf{D}[f[x],x]$ or $\mathbf{Derivative}[1][f]$. Each of these ways has its own context; details are discussed in Mathematica's Help.

Task

1. Type the command lines in the left-hand column below into Mathematica in the order in which they are listed. These command lines compute the first derivative of $f(x) = 1/(x^2 + 4)$ two ways, and then plot f and its first three derivatives. The effect of each command is described in the right-hand column for your reference.

In[1] := (* Your name, today's date *)	
In[2] := (* Differentiation *)	
In[3] := f[x_] := 1/(x^2+4)	Let $f(x) = 1/(x^2 + 4)$.
In[4] := (f[x+h]-f[x])/h	Set up the difference quotient.
In[5] := Expand[%]	Simplify the difference quotient.
In[6] := Together[%]	
In[7] := Limit[% , h->0]	Find the limit of the difference quotient as $h \rightarrow 0$.
In[8] := f'[x]	Find f' using Mathematica's built-in rules.
In[9] := Plot[{f[x], f'[x]}, {x,-4,4}]	Plot the graphs of f and f' .
In[10] := f''[x]	Find f'' .
In[11] := f'''[x]	Find f''' .
In[12] := Plot[{f[x], f'[x], f''[x], f'''[x]}, {x,-4,4}]	Plot the graphs of f , f' , f'' and f''' .

At this time make a hard-copy of your typed input and Mathematica's responses. Then:

2. On the first graphic you created in Task 1, label the graphs of f and f' , and highlight that part of the graph of f over which the tangent lines (to the graph of f) have positive slope, and that part of the graph of f' over which f' is positive.
3. On the second graphic you created in Task 1 label the graphs of f , f' , f'' and f''' .

Your lab report will be a hard copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).