The First Derivative and Slope

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Objective

To investigate the connection between the first derivative of a function f and the slope of the tangent line to the graph of f.

Narrative

Recall that, as long as the limit exists, the first derivative f' of the function f is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

In this project we use the definition of derivative to compute f' for $f(x) = 5x^3 + 2x^2 - 3x + 1$. We also graph f and two other functions and their derivatives to emphasize the connection between a function and its derivative: for each x in the domain of the derivative f' of f, $f'(x)$ is the slope of the tangent line to the graph of f at x, and f is increasing at x if and only if $f'(x) > 0$, and f is decreasing at x if and only if $f'(x) < 0$.

There are several different ways to write the derivative of a function in Mathematica. In this project we use the notation **f´** to denote the derivative of the function **f**. We could also write the derivative of **f** as **D[f[x],x]** or **Derivative[1][f]**. Each of these ways has its own context; details are discussed in Mathematica's Help.

Task

1. Type the command lines in the left-hand column below into Mathematica in the order in which they are listed. The effect of each command is described in the right-hand column for your reference.

`In[1]:=` **(* Your name, today´s date *)**	
`In[2]:=` **(* The First Derivative and Slope *)**	
`In[3]:=` **f[x_] := 5x^3+2x^2-3x+1**	Let $f(x) = 5x^3 + 2x^2 - 3x + 1$.
`In[4]:=` **(f[x+h]-f[x])/h**	Set up the difference quotient.
`In[5]:=` **Simplify[%]**	Simplify the difference quotient.
`In[6]:=` **Limit[%, h->0]**	Find the limit of the difference quotient as $h \to 0$.
`In[7]:=` **f´[x]**	Compute f'.
`In[8]:=` **Plot[{f[x], f´[x]}, {x,-4,4}]**	Plot the graphs of f and f'.
`In[9]:=` **f[x_] := x*Sin[x]^2**	Let $f(x) = x \sin^2 x$.
`In[10]:=` **Plot[{f[x], f´[x]}, {x,-4,4}]**	Plot the graphs of f and f'.
`In[11]:=` **f[x_] := x^2*Sin[x]**	Let $f(x) = x^2 \sin x$.
`In[12]:=` **Plot[{f[x], f´[x]}, {x,-4,4}]**	Plot the graphs of f and f'.

At this point, make a hard-copy of your typed input and Mathematica's responses. Then:

2. On each of the three graphics you created in Task 1, label the graphs of f and f'.

3. On each of the three graphics you created in Task 1, highlight that part of the graph of f over which the tangent lines (to the graph of f) have positive slope, and that part of the graph of f' over which f' is positive.

Your lab report will be a hard copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).