Newton's Method: Part 2

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Objective

To address some practical issues surrounding Newton's Method.

Narrative

In our study of Newton's Method we have, so far, neglected some practical issues surrounding the use of Newton's Method. One is that a function f often arises, you want to know one or all of its zeros, and no one is there to tell you if one exists, or — if one *does* exist — where to start looking for it. You can address this issue by graphing f using Mathematica. Another issue that arises involves the fact that Newton's Method doesn't necessarily converge if the initial estimate is not "good enough". In this project we address these issues.

Tasks

1. Try to find:

a) the zeroes of $f(x) = -2x^3 + 9x^2 - 11x + 3$ by going through at least 6 iterations of Newton's Method with an initial approximation of $x_1 = 1.0$. (Note how the approximations oscillate.)

b) the zero of $f(x) = x/(x^2 + 1)$ (which is clearly $x = 0$) by going through at least 6 iterations of Newton's Method with an initial approximation of $x_1 = 2.0$. (Note how the approximations get worse and worse the more times Newton's Method is iterated.)

You could address the issues raised above by choosing x_1 adequately close to the desired zero (which you can do by looking at the graph of f), and watching your approximations in an effort to detect either oscillation or divergence. You do not need to do this here, however, since our goal in this part of this project was simply to illustrate some of the practical issues that arise in using Newton's Method.

2. Use Newton's Method to find *all* solutions to the equation $\sin x = x^2 - 1$ to 8 decimal places of accuracy. (Start by graphing $\sin x$ and $x^2 - 1$ on the same set of coordinate axes to get a rough idea about how many solutions there are and where they are. For each solution, make an initial estimate, and apply Newton's Method to $f(x) = \sin x - (x^2 - 1)$.)

For each of these tasks, your lab report will be a hard copy of your typed input and Mathematica's responses (both text and graphics). Your response to Task 2 should include a summary statement of the form, "The solutions are ... ."