

Maximum and Minimum Values on a Closed Interval

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Objective

To illustrate how to find the maximum and minimum values of a function on a closed interval.

Narrative

The critical numbers of a function f on the interval (a, b) are the values of $x \in (a, b)$ for which $f'(x) = 0$ and $f'(x)$ does not exist. To find the absolute maximum and minimum values of a function f on a closed interval $[a, b]$ we first find the critical numbers of f on the open interval (a, b) , and then compare the values of f at these points with the values $f(a)$ and $f(b)$ of f at the endpoints. In this project we illustrate this process as we find the absolute maximum and minimum values of $f(x) = x/(x^2 + 1)$ on the interval $[-4, 4]$.

Tasks

1. Type the command lines in the left-hand column below into Mathematica in the order in which they are listed. The effect of each command is described in the right-hand column for your reference.

In[1] := (* Your name, today's date *)	
In[2] := (* Maximum and Minimum Values on a Closed Interval *)	
In[3] := f[x_] := x/(x^2+1)	Let $f(x) = x/(x^2 + 1)$.
In[4] := f'[x]	Here's $f'(x)$. Observe that it's a quotient.
In[5] := f1=Simplify[%]	Simplify $f'(x)$.
In[6] := Solve[Numerator[f1]==0,x]	Find where the numerator of $f'(x) = 0$.
In[7] := Solve[Denominator[f1]==0,x]	Find where the denominator of $f'(x) = 0$.

(In the case of $f(x) = x/(x^2 + 1)$, we didn't really have to use Mathematica to try to find where the denominator of $f'(x) = 0$ since it's clearly never 0. We included this line in the above code, however, to illustrate what you would need to do if you wish to apply the above code to another function whose denominator is not so easy to analyze.)

2. Use Mathematica to evaluate f at each critical number as well as at the endpoints -4 and 4 of the interval $[-4, 4]$.

3. Use Mathematica to draw the graph of f over the interval $[-4, 4]$.

At this time make a hard-copy of your typed input and Mathematica's responses. Then:

4. By hand, plot and label the points $P(c, f(c))$ where c is a critical number of f , as well as the points $A(a, f(a))$ and $B(b, f(b))$, on the graphic you created in Task 3.

5. Underneath your labeled graphic, state the maximum and minimum values of f on $[-4, 4]$. (A statement of the form, “___ is the maximum value for f , and ___ is the minimum value for f .” will suffice.)

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).