The Mean Value Theorem

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Objective

To illustrate the Mean Value Theorem.

Narrative

The Mean Value Theorem states that if f is continuous on $[a, b]$ and differentiable on (a, b), then there is at least one value of $c \in (a, b)$ for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

In this project we apply the Mean Value Theorem to $f(x) = x^4 - 16x^3 + 92x^2 - 224x + 200$ on $[1.6, 6.5]$.

Task

1. Type the command lines in the left-hand column below into Mathematica in the order in which they are listed. They plot the graph of f and the line L determined by the points $P(1.6, f(1.6))$ and $Q(6.5, f(6.5))$.

`In[1]:=` **(* Your name, today´s date *)**	
`In[2]:=` **(* The Mean Value Theorem *)**	
`In[3]:=` **f[x_] := x^4-16x^3+92x^2-224x+200**	Let $f(x) = x^4 - 16x^3 + 92x^2 - 224x + 200$.
`In[4]:=` **Plot[f[x], {x,0,8}, AxesOrigin -> {0, 0}]**	Graph of f over the interval $[0, 8]$.
`In[4]:=` **m = (f[6.5]-f[1.6])/(6.5-1.6)**	Let m denote the slope of L.
`In[5]:=` **Plot[{f[x], m(x - 1.6) + f[1.6]}, {x, 0, 8}, AxesOrigin->{0, 0}, PlotRange->{0, 25}]**	Graph f and L over the interval $[0, 8]$.

At this time make a hard-copy of your typed input and Mathematica's responses. Then:

2. Using a straightedge, draw the tangent lines to the graph of f at those points whose x-coordinates are between $x = 1.6$ and $x = 6.5$, that are parallel to L.

3. Using a straightedge, drop perpendiculars from the points of tangency of the tangents you drew in Task 1, to the x-axis, and plot, estimate (by eye) and label the values of c (along the x-axis) for which

$$f'(c) = \frac{f(6.5) - f(1.6)}{6.5 - 1.6}.$$

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).

Comments

The Mean Value Theorem plays a very important role in Calculus. From it, for example, follow the facts that:

1. a function f is increasing over an open interval I if and only if $f'(x) > 0$ for each $x \in I$, and f is decreasing over I if and only if $f'(x) < 0$ for each $x \in I$, and
2. if $D_x(f(x)) = D_x(g(x))$ then $f(x) = g(x) + C$ for some constant C.

The first of these facts is important in applying differentiation to curve sketching. The second is important in proving and applying the Fundamental Theorem of Calculus!