

Finding Relative Extrema on an Open Interval: The First Derivative Test

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Objective

To illustrate how to find and classify the relative extrema of a function on an open interval using the First Derivative Test.

Narrative

Recall that to find the maximum and minimum values of any function $f(x)$ on an open interval using the First Derivative Test we:

1. find the critical numbers of f (the values of x for which $f'(x) = 0$ and $f'(x)$ does not exist),
2. compute the sign of the first derivative f' of f at points x_0 and x_1 just to the left and right of each critical number c ($x_0 < c < x_1$): if
 - (a) $f'(x_0) > 0$ and $f'(x_1) < 0$ then $f(c)$ is a relative maximum,
 - (b) $f'(x_0) < 0$ and $f'(x_1) > 0$ then $f(c)$ is a relative minimum,
 - (c) $f'(x_0) > 0$ and $f'(x_1) > 0$, or $f'(x_0) < 0$ and $f'(x_1) < 0$, then $f(c)$ is neither a relative maximum nor a relative minimum. (In this last case, the point $P(c, f(c))$ is a saddle point.)

In this project we illustrate this process as we find the relative maximum and minimum values of $f(x) = 3x^4 - 16x^3 + 18x^2$.

Tasks

1. a) Type the command lines in the left-hand column below into Mathematica in the order in which they are listed. The effect of each command is described in the right-hand column for your reference.

In[1] := (* Your name, today's date *)

In[2] := (* The First Derivative Test *)

In[3] := f[x_] := 3x^4-16x^3+18x^2

In[4] := f'[x]

In[5] := Solve[f'[x]==0,x]

Let $f(x) = 3x^4 - 16x^3 + 18x^2$.

Here's $f'(x)$.

Find the critical numbers of f by solving the equation $f'(x) = 0$.

- b) Use Mathematica to evaluate f' at points x_0 and x_1 just to the left and right of each critical number.
- c) On the basis of your computations, write Mathematica comments stating whether $f(c)$ is a relative maximum or relative minimum, or whether $P(c, f(c))$ is a saddle point, for each critical number c . (Your comment might look like, “(* is a relative maximum *)” for example.)
- d) Type the following command line into Mathematica, substituting for **a** and **b** numbers that are large enough to enclose all the critical numbers of f and the graph of f over $[a, b]$.

In[6] := Plot[f[x],{x,a,b}]

At this time make a hard-copy of your typed input and Mathematica's responses. Then:

2. By hand, on the graphic you produced in Task 1, label the graph of f , plot and label the critical numbers of f along the x -axis, and plot and label the points $P(c, f(c))$, c a critical number of f . Remembering that the relative extrema of f are y -values, estimate the relative extrema of f by eye and for each relative

extremum write a sentence under your graphic of the form, “ f has a relative maximum of $_$ when $x = _$ ” or “ f has a relative minimum of $_$ when $x = _$ ”.

Your lab report will be a hard-copy of your typed input and Mathematica’s responses (both text and hand-labeled graphics).

Comments

To find the critical numbers of f we should find where $f'(x) = 0$ and where $f'(x)$ does not exist. We restricted our attention to where $f'(x) = 0$ in the above example since $f(x)$ is a polynomial function so $f'(x)$ exists for all x .