Finding Relative Extrema on an Open Interval: The First Derivative Test

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Objective

To illustrate how to find and classify the relative extrema of a function on an open interval using the First Derivative Test.

Narrative

Recall that to find the maximum and minimum values of any function $f(x)$ on an open interval using the First Derivative Test we:

1. find the critical numbers of f (the values of x for which $f'(x) = 0$ and $f'(x)$ does not exist),
2. compute the sign of the first derivative f' of f at points x_0 and x_1 just to the left and right of each critical number c ($x_0 < c < x_1$): if
   (a) $f'(x_0) > 0$ and $f'(x_1) < 0$ then $f(c)$ is a relative maximum,
   (b) $f'(x_0) < 0$ and $f'(x_1) > 0$ then $f(c)$ is a relative minimum,
   (c) $f'(x_0) > 0$ and $f'(x_1) > 0$, or $f'(x_0) < 0$ and $f'(x_1) < 0$, then $f(c)$ is a neither a relative maximum nor a relative minimum. (In this last case, the point $P(c, f(c))$ is a saddle point.)

In this project we illustrate this process as we find the relative maximum and minimum values of $f(x) = 4x^2/(x^4 + 8)$.

Tasks

1. a) Type the command lines in the left-hand column below into Mathematica in the order in which they are listed. The effect of each command is described in the right-hand column for your reference.

`In[1]:=` **(* Your name, today's date *)**	
`In[2]:=` **(* The First Derivative Test *)**	
`In[3]:=` **f[x_] := 4x^2/(x^4+8)**	Let $f(x) = 4x^2/(x^4 + 8)$.
`In[4]:=` **f´[x]**	Here's $f'(x)$. Observe that it's a quotient.
`In[5]:=` **f1=Simplify[%]**	Simplify $f'(x)$.
`In[6]:=` **Solve[Numerator[f1]==0,x]**	Find where the numerator of $f'(x) = 0$.
`In[7]:=` **Solve[Denominator[f1]==0,x]**	Find where the denominator of $f'(x) = 0$.

b) Use Mathematica to evaluate f' at points x_0 and x_1 just to the left and right of each critical number.

c) On the basis of your computations, write Mathematica comments stating whether $f(c)$ is a relative maximum or relative minimum, or whether $P(c, f(c))$ is a saddle point, for each critical number c. (Your comment might look like, "**(* _ is a relative maximum*)**" for example.)

d) Type the following command line into Mathematica, substituting for `a` and `b` numbers that are large enough to enclose all the critical numbers of f and the graph of f over $[a, b]$.

`In[8]:=` **Plot[f[x],{x,a,b}]**

At this time make a hard-copy of your typed input and Mathematica's responses. Then:

2. By hand, on the graphic you produced in Task 1, label the graph of f, plot and label the critical numbers of f along the x-axis, and plot and label the points $P(c, f(c))$, c a critical number of f. Remembering that the relative extrema of f are y-values, estimate the relative extrema of f by eye and for each relative extremum write a sentence under your graphic of the form, "f has a relative maximum of _ when $x =$ _" or "f has a relative minimum of _ when $x =$ _".

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).