

# The Second Derivative and Concavity

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## Objective

To illustrate the connection between the second derivative  $f''$  of a function  $f$  and the concavity of the graph of  $f$ .

## Narrative

Recall that the graph of a function  $f$  is concave up at  $x$  if  $f''(x) > 0$  and concave down at  $x$  if  $f''(x) < 0$ . In this project we illustrate this connection for  $f(x) = x^4 - 4x^3$ .

## Task

1. Type the command lines in the left-hand column below into Mathematica in the order in which they are listed. The effect of each command is described in the right-hand column for your reference.

In[1] := (\* Your name, today's date \*)

In[2] := (\* The Second Derivative and Concavity \*)

In[3] := f[x\_] := x^4-4x^3

Let  $f(x) = x^4 - 4x^3$ .

In[4] := f'[x]

Here's  $f'(x)$ .

In[5] := f''[x]

And here's  $f''(x)$ .

In[6] := Plot[{f[x], f''[x]},{x,-1,4}]

Plot the graphs of  $f$  and  $f''$  over  $[-1, 4]$ .

At this time make a hard-copy of your typed input and Mathematica's responses. Then:

2. By hand, label the graphs of  $f$  and  $f''$  in the graphic you produced in Task 1.

3. By hand, highlight that part of the graph of  $f$  over which  $f$  is concave up and that part of the graph of  $f''$  over which  $f''$  is positive.

Your lab report will be a hard copy of your typed input and Mathematica's responses (both text and graphics).