

Concavity and Inflection Points

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Objective

To illustrate how to analyze the concavity and find the inflection points of a function.

Narrative

The graph of a function f is concave up at x if $f''(x) > 0$ and concave down at x if $f''(x) < 0$, and an *inflection point* of f is a point $P(x, f(x))$ on the graph of f at which the concavity of f changes. To find the inflection points of f , we first find the values of x for which $f''(x) = 0$; these are the *potential inflection numbers* of f . We then look at the value of the second derivative just to the left and just to the right of each potential inflection number x : if the values have different signs then x is an *inflection number*, and $P(x, f(x))$ is an *inflection point* of f ; if the values have the same sign then x is not an inflection number, and $P(x, f(x))$ is not an inflection point of f .

In this project we analyze the concavity and find the inflection points of $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$.

Task

1. Type the command lines in the left-hand column below into Mathematica in the order in which they are listed. The effect of each command is described in the right-hand column for your reference.

In[1] := (* Your name, today's date *)	
In[2] := (* Concavity and Inflection Points *)	
In[3] := f[x_] := (x^4+x^2+1)/(x^4+1)	Let $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$.
In[4] := f'[x]	Here's $f'(x)$. Observe that it's a quotient.
In[4] := f''[x]	Here's $f''(x)$. Observe that it's also a quotient.
In[5] := f2 = Simplify[%]	Simplify $f''(x)$.
In[6] := Solve[Numerator[f2]==0,x]	Find where the numerator of $f''(x) = 0$.
In[7] := Solve[Denominator[f2]==0,x]	Find where the denominator of $f''(x) = 0$.

b) Use Mathematica to evaluate f'' at points x_0 and x_1 just to the left and right of each potential inflection number of f .

c) On the basis of your computations, write Mathematica comments stating whether $P(c, f(c))$ is an inflection point, for each potential inflection number c . (Your comment might look like, “(* ____ is an inflection point *)” for example.)

d) Type the following command line into Mathematica, substituting for **a** and **b** numbers that are large enough to enclose all the potential inflection numbers of f over $[a, b]$.

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In[8] := Plot[f[x],{x,a,b}]
In[9] := Plot[{f[x], f''[x]}, {x,a,b}, PlotStyle->{RGBColor[1,0,0],RGBColor[0,1,0]}
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At this time make a hard-copy of your typed input and Mathematica's responses. Then:

2. By hand, on the first graphic you produced in Task 1, label the graph of f , plot the potential inflection numbers of f along the x -axis, and plot and label the points $P(c, f(c))$ for c a potential inflection number of f . For each inflection *point* write a sentence under your graphic of the form, “____ is an inflection point”.

3. By hand, on the second graphic you produced in Task 1, label the graphs of f and f'' . Then highlight that part of the graph of f which is concave up, and that part of the graph of f'' over which f'' is positive.

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).