Critical Numbers and Inflection Points

Michael Penna, Indiana University – Purdue University, Indianapolis

Objective

To illustrate how Mathematica can be used to help you do your curve sketching homework.

Narrative

One of the major difficulties in curve sketching is finding where the derivatives of a function are zero and where they do not exist. In this project we illustrate how Mathematica can be used to assist in performing these tasks.

Task

1. Type the command lines in the left-hand column below into Mathematica in the order in which they are listed. These commands are aimed at finding where $f(x) = x^2(1-x^2)^{2/3}$ and its first two derivatives are zero and where they do not exist. The effect of each command is described in the right-hand column for your reference. Observe that we use Mathematica's **NSolve** command — rather than the **Solve** command — to generate numerical approximations to intercepts, critical numbers, and potential inflection numbers.

`In[1]:=` **(* Your name, today´s date *)**	
`In[2]:=` **(* Critical Numbers and Inflection Points *)**	
`In[3]:=` **f[x_] := x^2((1-x^2)^2)^(1/3)**	Let $f(x) = x^2(1-x^2)^{2/3}$.
`In[4]:=` **NSolve[f[x]==0,x]**	Find where $f(x) = 0$.
`In[5]:=` **f´[x]**	Here's $f'(x)$. Observe that it's a quotient.
`In[4]:=` **f1 = Simplify[%]**	Simplify $f'(x)$.
`In[3]:=` **NSolve[Numerator[f1]==0,x]**	Find where $f'(x) = 0$.
`In[8]:=` **NSolve[Denominator[f1]==0,x]**	Find where $f'(x)$ does not exist.
`In[9]:=` **f´´[x]**	Here's $f''(x)$. Observe that it's also a quotient.
`In[10]:=` **f2 = Simplify[%]**	Simplify $f''(x)$.
`In[11]:=` **NSolve[Numerator[f2]==0,x]**	Find where $f''(x) = 0$.
`In[12]:=` **NSolve[Denominator[f2]==0,x]**	Find where $f''(x)$ does not exist.
`In[13]:=` **Plot[f[x], {x,-4,4}]**	Plot the graph of f.

The reason we wrote $x^2(1-x^2)^{2/3}$ as **x^2((1-x^2)^2)^(1/3)** in the above code is that when $|x| > 1$, $1-x^2 < 0$ and in this case Mathematica computes $(1-x^2)^{2/3}$ by taking the cube root of $1-x^2$ — which it views as a complex number — and squaring — yielding another complex number (which is something we don't want: in this course we're restricting our attention to the real numbers). In writing **((1-x^2)^2)^(1/3)** we are forcing Mathematica to square $1-x^2$ first — yielding a real number — and then take the cube root — yielding another real number.

At this time make a hard-copy of your typed input and Mathematica's responses. Then:

2. On the graphic you created, plot and label the points along the x-axis at which $f(x) = 0$, at which $f'(x) = 0$, at which $f'(x)$ does not exist, at which $f''(x) = 0$, and at which $f''(x)$ does not exist. Label a point at which $f(x) = 0$ as an intercept, a point at which $f'(x) = 0$ or $f'(x)$ does not exist as a critical number, and a point at which $f''(x) = 0$ or $f''(x)$ does not exist as a potential inflection number.

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).