Graphing Functions

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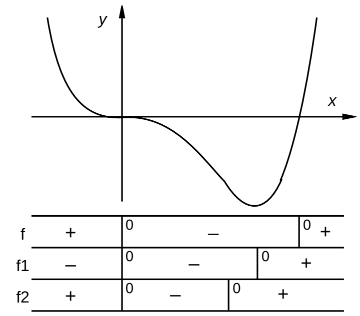
Objective

To investigate what we can say about the graph of a function f given where f and its first and second derivatives are positive, negative, and zero.

Narrative

To record where a function f and its first and second derivatives f' and f'', respectively, are positive, negative, and zero, we use three "recording strips" below the graph of f as illustrated in the figure at the right. Study this figure carefully. Note how the vertical bars are drawn where f, f', and f'' are 0, and the spaces between the vertical bars are labelled + or − depending on the sign of f, f', and f'' over those intervals.



Tasks

1. Type the command lines below into Mathematica in the order in which they are listed. They produce a graph of the function $f(x) = 4x/(1 + x^2)$, and three recording strips below the graph of f.

```
In[1]:= (* Your name, today´s date *)
In[2]:= (* Graphing Functions *)
In[3]:= (* Task 1 *)
In[4]:= f[x_] := 4x/(1+x^2)
In[5]:= Plot[{f[x],-3.5,-4.5,-5.5,-6.5}, {x,-6,6}, PlotRange->{-6.5,3}, Ticks->None]
```

2. a) Type the command lines below into Mathematica in the order in which they are listed. They determine where $f(x) = x^2(1 - x)^{2/3}$ and its first and second derivatives are zero; this information will be used later in this project.

```
In[6]:= (* Task 2a *)
In[7]:= f[x_] := x^2((1-x^2)^2)^(1/3))
In[8]:= NSolve[f[x]==0,x]
In[9]:= f´[x]
In[10]:= f1=Simplify[%]
In[11]:= NSolve[Numerator[f1]==0,x]
In[12]:= NSolve[Denominator[f1]==0,x]
In[13]:= f´´[x]
In[14]:= f2=Simplify[%]
In[15]:= NSolve[Numerator[f2]==0,x]
In[16]:= NSolve[Denominator[f2]==0,x]
```

b) Type the command line below into Mathematica. It produces an empty graph and three recording strips.

```
In[17]:= (* Task 2b *)
In[18]:= Plot[{-3.5,-4.5,-5.5,-6.5}, {x,-6,6}, PlotRange->{-6.5,3}, Ticks->None]
```

At this time make a hard-copy of your typed input and Maple's responses. (But don't shut down Mathematica yet! It will be very helpful in what's to come!) Then:

3. Fill in the recording strips below the graphic you produced in Task 1 with +'s and −'s using the graph of f as a guide.

4. a) Fill in the recording strips on the graphic you produced in Task 2(b) using the information you computed in Task 2(a) as a guide. (You will need to test the values of f, f', and f'' between their respective zeroes to determine where they are positive and negative. The reason you were advised not to shut down Mathematica earlier was that you can now use Mathematica to easily evaluate f, f', and f''!)

b) Use the information you recorded in part (a) of this task to sketch the graph of f in the space provided.

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).

Comments

To graph an arbitrary function f we need to determine not only where f and its first and second derivatives f' and f'' are positive, negative, and zero, but also where they do not exist.

The points where:

1. $f(x) = 0$ are the x-intercepts of the graph of f,
2. $f(x)$ does not exist are the x-intercepts of the vertical asymptotes of the graph of f,
3. $f'(x) = 0$ are critical points of the graph of f,
4. $f'(x)$ does not exist are the x-intercepts of the vertical tangents to the graph of f,
5. $f''(x) = 0$ and $f''(x)$ do not exist are the x-intercepts of the possible inflection points of the graph of f.

For a function f for which $f(x)$, $f'(x)$, and/or $f''(x)$ do not exist for certain values of x, we draw and label vertical bars in our recording strips with $\pm\infty$.