# Newton's Method

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***Objective***

To study Newton's Method.

***Narrative***

Newton's Method is a method for finding an approximation to a value of $x$ for which $f(x) = 0$ (or a *zero* of $f$) given an initial approximation $x_1$ to $x$. This method uses the iterative equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \ldots,$$

$x_1$ being given, to obtain successive approximations $x_2$, $x_3$, ..., to $x$.

***Task***

**1**. a) Type the command lines in the left-hand column below into Mathematica in the order in which they are listed. These commands apply Newton's Method to finding the root of $f(x) = x^2 - 2 = 0$ given an initial estimate of $x_1 = 1.0$. Note that Mathematica uses brackets [ ] to denote subscripts.

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In[1]:= (* Your name, today's date *)
In[2]:= (* Newton's Method *)
In[3]:= f[x_] := x^2-2
In[4]:= x[1] = 1.0
In[5]:= For[n=1, n<10, n++,
            {x[n+1] = x[n]-f[x[n]]/f´[x[n]];
             Print[n+1, "    ", N[x[n+1],10]];}]
In[6]:= Plot[f[x], {x,0,2}]
```

At this time make a hard-copy of your typed input and Mathematica's responses. Then:

b) To 8 decimal places of accuracy, what number $x_\infty$ does it appear that $x_n$ is converging to?

c) On the graphic you created in part (a), plot and label the points $P_1, P_2, P_3$, and $P_\infty$ on the $x$-axis whose $x$-coordinates are $x_1, x_2.x_3$, and $x_\infty$.

**2**. Use Newton's Method to estimate the solutions to the equation $\sin x = x^2 - 1$ by proceeding as follows:

a) Plot the graph of $f(x) = \sin x - (x^2 - 1)$ for $x \in [-5, 5]$. (This graph will help you estimate the roots of $f(x) = 0$.)

b) Estimate each zero of $f(x)$ by a number $x_1$, and iterate Newton's Method enough times to obtain an approximation to the root to 8 decimal places of accuracy. (Your initial estimate of each root must be close enough to the root for Newton's Method to converge: if it's not close enough, the values $x_1$, $x_2$, $x_3$, ... might not converge to the value you're interested in!)

At this time again make a hard-copy of your typed input and Mathematica's responses. Then:

c) To 8 decimal places of accuracy, what number $x_\infty$ does it appear that $x_n$ is converging to?

d) On the graphic you created in part (b), plot and label the points $P_1, P_2, P_3$, and $P_\infty$ on the $x$-axis, whose $x$-coordinates are $x_1, x_2, x_3$, and $x_\infty$.

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).