Ballistics

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Objective

To illustrate an important application of differentiation to ballistics.

Narrative

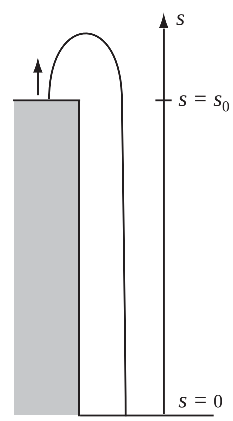
If a projectile is fired vertically upward with an initial velocity of v_0 ft/sec from an initial position s_0 feet above the ground (see the figure to the right), then (neglecting air resistance) after t sec the projectile is

$$s = s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$$

feet above the ground, where $g = 32$ ft/sec^2 is acceleration due to gravity, and the velocity of the projectile is

$$v = v(t) = D_t(s(t)) = -gt + v_0$$

meters per second. (If we use metric units then $g = 9.8$ m/sec^2.)



Tasks

1. Type the command lines in the left-hand column below into Mathematica in the order in which they are listed.

`In[1]:=` **(* Your name, today's date *)**	
`In[2]:=` **(* Ballistics *)**	
`In[3]:=` **{g = 32, t0 = 0, s0 = 100, v0 = 128}**	Let $g = 32, t_0 = 0, s_0 = 100$, and $v_0 = 128$.
`In[4]:=` **s[t_] = -0.5*g*t^2+v0*t+s0**	Let $s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$,
`In[5]:=` **v[t_] = s´[t]**	and $v(t) = D_t(s(t))$.
`In[6]:=` **{t1=1.0, s[t1]}**	Let $t_1 = 1.0$, and compute $s(t_1)$.

2. By using trial-and-error, change the value of **t1** in **In[6]** until you obtain the value of **t1** *greater than* **t0** for which **s[t1]** is within 2 decimal places of 0.

3. Continue by typing the command lines in the left-hand column below into Mathematica in the order in which they are listed. (The **In**-line numbers below may not agree with yours.)

`In[7]:=` **Plot[s[t], {t,t0,t1}]**	Plot s as a function of t for $t \in [t_0, t_1]$.
`In[8]:=` **v[t1]**	Find the velocity $v(t_1)$ with which the projectile strikes the ground.
`In[9]:=` **Solve[v[t]==0,t]**	See below.
`In[10]:=` **t /. Solve[v[t]==0,t]**	Again, see below.
`In[11]:=` **tmax = %[[1]]**	The time t_{max} is the time at which the velocity of the projectile is 0 (the time at which the projectile has reached its maximum altitude).
`In[12]:=` **s[tmax]**	Find the maximum altitude $s(t_{max})$ of the projectile.

The effect of the code, "**Solve[v[t]==0,t]**" in **In[10]** is to solve the equation $v(t) = 0$ for t; the result is a list L containing the rule $t \to 4$ that associates to t the value 4. (If the equation $v(t) = 0$ had more than one solution then L would have more than one entry.) The effect of the code, "**t /. Solve[v[t]==0,t]**" is to create a list whose entry is the value obtained by applying the rule contained in L to **t**. (If L had more than one entry then the result would be the list obtained by applying each rule to **t**.)

At this time make a hard-copy of your typed input and Mathematica's responses. Then:

4. Label the coordinate axes in the graphic you produced by hand (one should be a t-axis, and the other an s-axis), as well as the location of the projectile at times $t = t_0$, $t = t_1$, and $t = t_{max}$.

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).

Comments

In this project we found the time it takes a projectile fired vertically upward from 100 feet above the ground at an initial velocity of 128 ft/sec, to hit the ground, the velocity with which it hits the ground, the time it takes to achieve its maximum altitude, and its maximum altitude. A slightly different problem involves analyzing the motion of a projectile fired at an elevation angle θ above the horizontal, with an initial velocity of v_0 ft/sec from a point s_0 feet above the ground. A graphic illustrating such a situation can be produced with the following Mathematica code:

```
In[1]:= (* Parametrized Curve *)
In[2]:= {g = 32, t0 = 0, s0 = 100, v0 = 128}
In[3]:= Theta = Pi/4
In[4]:= x[t_] := v0*Cos[Theta]*t
In[5]:= y[t_] := -0.5*g*t^2+v0*Sin[Theta]*t+s0
In[6]:= t1 = 8.71695
In[7]:= y[t1]
In[8]:= ParametricPlot[{x[t],y[t]}, {t,t0,t1}]
```

In this graphic, however, the horizontal axis is an x-axis, the vertical axis is a y-axis, and there is no t-axis! Thus there is a basic difference between what is being illustrated by this graph, and what was illustrated by the graph you drew earlier in this project.