

Ballistics

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Objective

To illustrate an important application of differentiation to ballistics.

Narrative

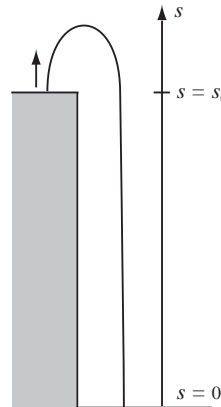
If a projectile is fired vertically upward with an initial velocity of v_0 m/sec from an initial position s_0 meters above the ground (see the figure to the right), then (neglecting air resistance) after t sec the projectile is

$$s = s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$$

meters above the ground, where $g = 9.8$ m/sec² is acceleration due to gravity, and the velocity of the projectile is

$$v = v(t) = D_t(s(t)) = -gt + v_0$$

meters per second. (If we use British units then $g = 32$ ft/sec².)



Tasks

1. Type the command lines in the left-hand column below into Mathematica in the order in which they are listed.

In[1] := (* Your name, today's date *)	
In[2] := (* Ballistics *)	
In[3] := {g = 9.8, t0 = 0, s0 = 100, v0 = 128}	Let $g = 9.8, t_0 = 0, s_0 = 100,$ and $v_0 = 128.$
In[4] := s[t_] = -0.5*g*t^2+v0*t+s0	Let $s(t) = -\frac{1}{2}gt^2 + v_0t + s_0,$
In[5] := v[t_] = s'[t]	and $v(t) = D_t(s(t)).$
In[6] := {t1=1.0, s[t1]}	Let $t_1 = 1.0,$ and compute $s(t_1).$

2. By using trial-and-error, change the value of **t1** in **In[6]** until you obtain the value of **t1** greater than **t0** for which **s[t1]** is within 2 decimal places of 0.

3. Continue by typing the command lines in the left-hand column below into Mathematica in the order in which they are listed. (The **In**-line numbers below may not agree with yours.)

In[7] := Plot[s[t], {t,t0,t1}]	Plot s as a function of t for $t \in [t_0, t_1].$
In[8] := v[t1]	Find the velocity $v(t_1)$ with which the projectile strikes the ground.
In[9] := Solve[v[t]==0,t]	See below.
In[10] := t /. Solve[v[t]==0,t]	Again, see below.
In[11] := tmax = %[[1]]	The time t_{max} is the time at which the velocity of the projectile is 0 (the time at which the projectile has reached its maximum altitude).
In[12] := s[tmax]	Find the maximum altitude $s(t_{max})$ of the projectile.

The effect of the code, “**Solve[v[t]==0,t]**” in **In[10]** is to solve the equation $v(t) = 0$ for t ; the result is a list L containing the rule $t \rightarrow 4$ that associates to t the value 4. (If the equation $v(t) = 0$ had more than one solution then L would have more than one entry.) The effect of the code, “**t /. Solve[v[t]==0,t]**” is

to create a list whose entry is the value obtained by applying the rule contained in L to \mathbf{t} . (If L had more than one entry then the result would be the list obtained by applying each rule to \mathbf{t} .)

At this time make a hard-copy of your typed input and Mathematica's responses. Then:

4. Label the coordinate axes in the graphic you produced by hand (one should be a t -axis, and the other an s -axis), as well as the location of the projectile at times $t = t_0$, $t = t_1$, and $t = t_{max}$.

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).

Comments

In this project we found the time it takes a projectile fired vertically upward from 100 meters above the ground at an initial velocity of 128 m/sec, to hit the ground, the velocity with which it hits the ground, the time it takes to achieve its maximum altitude, and its maximum altitude. A slightly different problem involves analyzing the motion of a projectile fired at an elevation angle θ above the horizontal, with an initial velocity of v_0 m/sec from a point s_0 meters above the ground. A graphic illustrating such a situation can be produced with the following Mathematica code:

```
In[1] := (* Parametrized Curve *)
In[2] := {g = 9.8, t0 = 0, s0 = 100, v0 = 128}
In[3] := Theta = Pi/4
In[4] := x[t_] := v0*Cos[Theta]*t
In[5] := y[t_] := -0.5*g*t^2+v0*Sin[Theta]*t+s0
In[6] := t1 = 26.8816
In[7] := y[t1]
In[8] := ParametricPlot[{x[t],y[t]}, {t,t0,t1}]
```

In this graphic, however, the horizontal axis is an x -axis, the vertical axis is a y -axis, and there is no t -axis! Thus there is a basic difference between what is being illustrated by this graph, and what was illustrated by the graph you drew earlier in this project.