

The Fundamental Theorem of Calculus

Michael Penna, Indiana University – Purdue University, Indianapolis

Objective

To illustrate the Fundamental Theorem of Calculus.

Narrative

Recall that the Fundamental Theorem of Calculus states that if f is a continuous function on the finite closed interval $[a, b]$ then

$$D_x \left(\int_{t=a}^x f(t) dt \right) = f(x)$$

for $x \in (a, b)$, and hence

$$\int_{x=a}^b f(x) dx = F(b) - F(a)$$

where F is any antiderivative for f (that is, $F'(x) = f(x)$).

Task

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands are aimed at investigating the Fundamental Theorem of Calculus in the context of $\int_{x=1}^4 \frac{1}{x} dx$. The effect of each command is described in the right-hand column for your reference.

In[1] := (* Your name, today's date *)	
In[2] := (* The Fundamental Theorem of Calculus *)	
In[3] := f[x_] := 1/x	Let $f(x) = 1/x$.
In[4] := f[1]	Find $f(1)$.
In[5] := Plot[f[x], {x,-5,5}]	Graph f over the interval $[-5, 5]$.
In[6] := {a=1, b=4}	Let $a = 1$ and $b = 4$.
In[7] := Limit[Sum[f[a+i*(b-a)/n]*(b-a)/n, {i,1,n}], n->Infinity]	Find $\int_{x=a}^b f(x) dx$ thinking of it as an area.
In[8] := Integrate[f[x], {x,1,4}]	Find $\int_{x=a}^b f(x) dx$ using Mathematica.
In[9] := F[x_] := Integrate[f[t], {t,1,x}, Assumptions->x>1]	Let $F(x) = \int_{t=1}^x f(t) dt$. (Note that we're defining F as an integral.)
In[10] := Plot[{f[x], F[x]}, {x,1,4}, AxesOrigin->{0,0}]	Graph f and F over the interval $[1, 4]$.
In[11] := F'[x]	The Fundamental Theorem of Calculus states that $F' = f$.
In[12] := F[b]-F[a]	Find $F(b) - F(a)$. (This should equal $\int_{x=a}^b f(x) dx$.)

At this time make a hard-copy of your typed input and Maple's responses. Then:

2. Label the graphs of f and F in the plot you made in Task 1 by hand.
3. Plot and label the point $P(3, F(3))$.
4. The value $F(3)$ represents an area. Shade in the area that corresponds to $F(3)$. (Be careful! Think!)

Your lab report will be a hard-copy of your typed input and Maple's responses (both text and hand-labeled graphics).