

Areas of Type I and Type II Regions

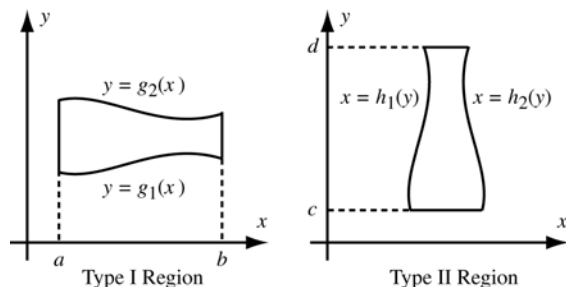
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Objective

To illustrate how to find the areas of Type I and Type II regions.

Narrative

There are two basic types of regions in the xy -coordinate plane: Type I regions and Type II regions. Type I regions are regions bounded by lines $x = a$ and $x = b$, where $a < b$, and curves $y = g_1(x)$ and $y = g_2(x)$, where $g_1(x) \leq g_2(x)$ for $a \leq x \leq b$. Type II regions are bounded by lines $y = c$ and $y = d$, where $c < d$, and curves $x = h_1(y)$ and $x = h_2(y)$, where $h_1(y) \leq h_2(y)$ for $c \leq y \leq d$. Every region in the xy -coordinate plane can be subdivided into regions of Type I and Type II, and hence to find the area of any region in the plane it suffices to know how to find the area of regions of Type I and Type II.



The area of the Type I and Type II regions described above are, respectively,

$$\int_{x=a}^b (g_2(x) - g_1(x)) dx \quad \text{and} \quad \int_{y=c}^d (h_2(y) - h_1(y)) dy.$$

Task

1. a) Type the command lines below into Mathematica in the order in which they are listed. They produce a graphic of the Type I region R bounded on the left by $x = 1$, on the right by $x = 3$, below by $y = 1/2 - x/6$, and above by $y = \sqrt{x}$ (using **ImplicitPlot** from Mathematica's **Graphics** package), and find the area of R .

```
In[1] := (* Your name, today's date *)
```

```
In[2] := (* Areas of Type I and Type II Regions *)
```

```
In[3] := <<Graphics`ImplicitPlot`
```

```
In[4] := (* Task 1a *)
```

```
In[5] := ImplicitPlot[{x==1, x==3, y==1/2-x/6, y==Sqrt[x]}, {x,0,4}, {y,0,2}]
```

```
In[6] := Integrate[Sqrt[x]-(1/2-x/6), {x,1,3}]
```

b) Repeat part (a) for the Type 1 region R bounded on the left by $x = -1$, on the right by $x = 2$, below by $y = x^2$, and above by $y = x + 3$. (Since **ImplicitPlot** has already been loaded, you do not need to load it again.)

2. a) Continue by typing the command lines below into Mathematica in the order in which they are listed. They produce a graphic of the Type II region R bounded on the bottom by $y = -2$, on the top by $y = 1$, on the left by $x = y - 1$, and on the right by $x = y^2$, and find the area of R .

```

In[10]:= (* Task 2a *)
In[11]:= ImplicitPlot[{y== -2, y==1, x==y-1, x==y^2}, {x,-4,4}, {y,-3,2}]
In[12]:= Integrate[y^2-(y-1), {y,-2,1}]

```

b) Repeat part (a) for the Type 2 region R bounded on the bottom by $y = -1$, on the top by $y = 2$, on the left by $x = y + 3$, and on the right by $x + 2 = y^2$.

At this time make a hard-copy of your typed input and Mathematica's responses. Then:

3. On each of the graphics you produced for Tasks 1 and 2, by hand:

- a) label each side of the region R with its equation, and
- b) shade in the region R .

Your lab report will consist of the figures you drew by hand, your typed input and Mathematica's responses (both text and hand-labeled graphics).

Comments

The following code uses Mathematica's **Line** function:

```
Line[{x0,y0},{x1,y1}]
```

draw a line from $P_0(x_0, y_0)$ to $P_1(x_1, y_1)$.

to draw the region discussed in Task 1a another way.

```

In[1]:= Plot1 =Plot[{1/2-x/6, Sqrt[x]}, {x,0,4}]
In[2]:= Plot2 = Graphics[{Line[{{1,0},{1,1}}, Line[{{3,0},{3,Sqrt[3]}]}]}]
In[3]:= Show[Plot2]
In[4]:= Show[{Plot1,Plot2}]

```