The Average Value of a Function

Michael Penna, Indiana University – Purdue University, Indianapolis

Objective

To investigate the average value of a function.

Narrative

Recall that by definition, the average value of the function $f = f(x)$ over the interval $[a, b]$ is

$$\frac{1}{b-a}\int_{x=a}^{b} f(x)\ dx.$$

Task

1. Type the command lines below into Mathematica in the order in which they are listed. These commands compute the average value of $f(x) = 18x - x^3$ over the interval $[1, 3]$.

```
In[1]:= (* Your name, today's date *)
In[2]:= (* The Average Value of a Function *)
In[3]:= f[x_] := 18x-x^3
In[4]:= {a=1, b=3}
In[5]:= Plot1 = Plot[f[x], {x,0,3Sqrt[2]}, AxesOrigin->{0,0}]
In[6]:= Avf = (1/(b-a))*Integrate[f[x], {x,a,b}]
In[7]:= Plot2 = Graphics[Line[{{a,0}, {a,Avf}, {b,Avf}, {b,0}}]]
In[8]:= Show[{Plot1,Plot2}]
```

At this point, make a hard-copy of your typed input and Mathematica's responses. Then:

2. There is a number $c \in [a, b]$ for which **f(c) = Avf**. Plot and label the point $x = c$ on the x-axis, draw by hand a vertical line at this point that meets the graph of f at the point $P(c, f(c))$, and lightly shade in by hand the rectangle whose area is $\int_{x=a}^{b} f(x)\ dx$.

3. Does such a c always exist? That is, if f is any function defined on a closed interval $[a, b]$, is there always a number $c \in [a, b]$ for which **f(c) = Avf**? If not always, under what conditions does such a c exist? Justify your answer.

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-drawn graphics).