Inverse Functions

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Objective

To investigate some properties of inverse functions.

Narrative

In this project we investigate some properties of inverse functions. Some of the key things you should know about inverse functions are:

1. To show that a function f has an inverse, you need to show that f is $1-1$. If f is differentiable then you can do this by showing that either: a) $f'(x) > 0$ for all x in the domain of f, or b) $f'(x) < 0$ for all x in the domain of f.
2. You can find the inverse f^{-1} of a simple function f by solving the equation $y = f(x)$ for x in terms of y; the resulting equation is $x = f^{-1}(y)$. (So if you are looking for $f^{-1}(x)$, simply interchange x and y in this equation.) Remember that this only works for simple functions f.
3. One way to check your work in computing the inverse f^{-1} of a simple function f is to verify that: a) $f^{-1}(f(x)) = x$ for all x in the domain of f, and b) $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}.
4. The graph of the inverse f^{-1} of a function f is the reflection of the graph of f in the line $y = x$.
5. The derivative of f is related to the derivative of f^{-1} by the equation $D_x(f^{-1}(x)) = 1/D_y(f(y))$.

Tasks

1. Type the command lines below into Mathematica in the order in which they are listed. The effect of these commands is to graph $f(x) = \sin x$ and $g(x) = f^{-1}(x) = \arcsin x$.

```
In[1]:= (* Your name, today´s date *)
In[2]:= (* Inverse Functions *)
In[3]:= Plot1 = Plot[Sin[x], {x,0,Pi/2}, AspectRatio->1]
In[4]:= Plot2 = Plot[ArcSin[x], {x,0,Pi/2}, AspectRatio->1]
In[5]:= Plot3 = Plot[x, {x,0,Pi/2}, AspectRatio->1]
In[6]:= Show[{Plot1, Plot2, Plot3}]
```

At this point, make a hard-copy of your typed input and Mathematica's responses (both text and graphics). Then:

2. Using a straightedge, very carefully draw the tangent line to the graph of f at the point $P_0(1, \sin 1)$ and the tangent line to the graph of f^{-1} at the point $Q_0(\sin 1, 1)$.

3. a) By eye, estimate the coordinates of a point P_1 other than P_0 on the tangent line to the graph of f at P_0 and the coordinates of a point Q_1 other than Q_0 on the tangent line to the graph of f^{-1} at Q_0.

b) Use P_0 and P_1 to estimate the slope of the tangent line to the graph of f at P_0, and Q_0 and Q_1 to estimate the slope of the tangent line to the graph of f^{-1} at Q_0.

4. a) What should the relationship between the two slopes you computed in Task 3(b) be?

b) Why should this relationship be true?

c) Numerically verify the relationship between the two slopes you computed in Task 3(b).

Your lab report will be a hard-copy of your typed input Mathematica's responses (both text and hand-drawn graphics), and your written responses.