

Logarithmic Identities and Inequalities

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Objective

To investigate various logarithmic identities and inequalities.

Narrative

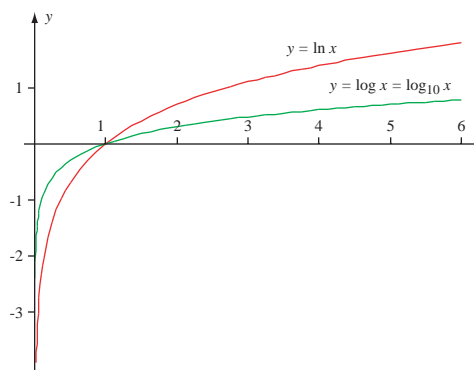
Throughout this course we will make use of various logarithmic identities and inequalities. In this project we investigate some of these identities and inequalities. To set the stage for this project, recall that:

$$\log_B(xy) = \log_B x + \log_B y$$

$$\log_B \frac{x}{y} = \log_B x - \log_B y$$

$$\log_B(x^k) = k \log_B x$$

$$\log_B x = \frac{\ln x}{\ln B}$$



Thus, for example,

$$\log_{10} x = \frac{\ln x}{\ln 10} = \frac{1}{\ln 10} \ln x \approx 0.4343 \ln x.$$

so the graph of $g(x) = \log_{10} x$ is a compression of the graph of $f(x) = \ln x$ by a factor of $\frac{1}{\ln 10} \approx 0.4343$ in the y -direction.

Task

1. a) Type the command lines below into Mathematica in the order in which they are listed. These commands produce plots of the graphs of $f(x) = \ln x$, $g(x) = \ln(2x)$, $h(x) = \ln(x/2)$, and $k(x) = \ln(x^2)$ over the interval $[0, 6]$ on one set of axes. *Note:* In Mathematica, the natural log function is written **Log[x]**, and the natural exponential function is written **Exp[x]**.

```
In[1] := (* Your name, today's date *)
```

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In[2] := (* Logarithmic Identities and Inequalities *)
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In[3] := Plot[{Log[x], Log[2x], Log[x/2], Log[x^2]}, {x,0,6}]
```

b) Continue by typing the command line below into Mathematica. It draws the graphs of $f(x) = \ln x$, $g(x) = (x-1)/x$, and $h(x) = x-1$ over the interval $[0, 6]$ on one set of axes.

```
In[4] := Plot[{Log[x], (x-1)/x, x-1}, {x,0.5,2}]
```

At this point, make a hard-copy of your typed input and Mathematica's responses (both text and graphics). Then:

2. a) Label the graphs of the functions in the plot you created in part (a) of Task 1 by hand.

b) Describe (in terms of vertical shifting, vertical stretching, and vertical reflecting) the relationship between the graphs of the following pairs of functions, and justify each answer with a logarithmic identity as we did in the Narrative.

i) $f(x) = \ln x$ and $g(x) = \ln(2x)$,

ii) $f(x) = \ln x$ and $h(x) = \ln(x/2)$,

iii) $f(x) = \ln x$ and $k(x) = \ln(x^2)$.

3. a) Label the graphs of the functions in the plot you created in part (b) of Task 1 by hand.
- b) What double inequality does the plot you created in part (b) of Task 1 suggest? (A *double inequality* is an inequality of the form $a \leq b \leq c$.)

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-drawn graphics).

Comments

The double inequality discussed in this project is useful in the discussion of sequences and series.