

Solutions of a Differential Equation

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Objective

To discuss the solutions of a differential equation in the context of antiderivatives (or indefinite integrals).

Narrative

For any differentiable function f we can compute $y' = \frac{dy}{dx} = f'(x)$; if we try to retrieve f by finding the antiderivative (or indefinite integral) of f' , however, we can only retrieve f up to an additive constant C . For any fixed value of C , the graph of this function is a vertical shift of the graph of f .

Similarly, for any equation $E(x, y) = 0$ (for which E is differentiable in x and y) we can compute $y' = \frac{dy}{dx}$; if we try to retrieve $E(x, y) = 0$ by solving a differential equation, however, we can only retrieve $E(x, y)$ up to an additive constant C . For any fixed value of C , the graph of this equation is a curve which is “parallel” to the graph of $E(x, y) = 0$.

In this project we use the command:

DSolve[*differential equation in x and y , y , x*] solves the differential equation in x and y for $y = y(x)$.

As illustrated below, in using **DSolve** we be careful in specifying y by “**y[x]**”.

Task

1. We begin by studying $y = f(x) = x^2 + e^x + 3$. Type the command lines below into Mathematica in the order in which they are listed. They differentiate f , and then integrate f' and graph several of its solutions.

In[1] := (* Your name, today's date *)

In[2] := (* Solutions of a Differential Equation *)

In[3] := (* Task 1 *)

In[4] := y[x_] = x^2+Exp[x]+3

In[5] := Plot[y[x], {x,-2,2}]

In[6] := y'[x]

In[7] := Clear[y]

In[8] := DSolve[y'[x]==2x+Exp[x], y, x]

Solve the differential equation

$$y' = 2x + e^x.$$

The solution in the case $C = 1$.

The solution in the case $C = 2$.

The solution in the case $C = 3$.

The solution in the case $C = 4$.

The solution in the case $C = 5$.

In[9] := y1[x_] = x^2+Exp[x]+1

In[10] := y2[x_] = x^2+Exp[x]+2

In[11] := y3[x_] = x^2+Exp[x]+3

In[12] := y4[x_] = x^2+Exp[x]+4

In[13] := y5[x_] = x^2+Exp[x]+5

In[14] := Plot[{y1[x], y2[x], y3[x], y4[x], y5[x]}, {x,-2,2}]

2. We now study the equation $x^2 + 3y^2 = 3$. Continue by typing the command lines below into Mathematica in the order in which they are listed. These commands compute $y' = \frac{dy}{dx}$, and then solve the resulting differential equation and graph several of its solutions.

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In[15] := (* Task 2 *)
In[16] := x^2+3y[x]^2 == 3
In[17] := <<Graphics`ImplicitPlot`
In[18] := ImplicitPlot[x^2+3y^2 == 3, {x,-2,2}, {y,-2,2}]
In[19] := D[x^2+3y[x]^2,x] == D[3,x]
In[20] := DSolve[2x+6y[x]y'[x] == 0, y, x]           Observe that we can combine both solutions
                                                    into the single solution  $x^2 + 3y^2 = C$ .
In[21] := eqn1 = x^2+3y^2 == 1                         The solution in the case  $C = 1$ .
In[22] := eqn2 = x^2+3y^2 == 2                         The solution in the case  $C = 2$ .
In[23] := eqn3 = x^2+3y^2 == 3                         The solution in the case  $C = 3$ .
In[24] := eqn4 = x^2+3y^2 == 4                         The solution in the case  $C = 4$ .
In[25] := eqn5 = x^2+3y^2 == 5                         The solution in the case  $C = 5$ .
In[26] := ImplicitPlot[{eqn1, eqn2, eqn3, eqn4, eqn5}, {x,-2,2}, {y,-2,2}]

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Observe that in Task 1 we are dealing with functions and that the graphs of antiderivatives are the graphs of functions, and that in Task 2 we are dealing with equations and that the graphs of the solutions to our differential equation are graphs of curves (which are *not* the graphs of functions).

At this time, make a hard-copy of your typed input and Mathematica's responses. Then, by hand:

3. a) Label the curves in the first graphics you produced in Tasks 1 and 2, with their equations by hand. (For example, label the graph of $f(x) = x^2 + e^x$ by " $f(x) = x^2 + e^x$ ".)

b) Label each of the curves in the second graphics you produced in Tasks 1 and 2, with the corresponding value of C by hand. (For example, label each curve corresponding to $C = 1$ by " $C = 1$ ".)

4. Plot and label the point $P(1, 0.5)$ in the second graphics you produced in Tasks 1 and 2, and draw the solution curves that pass through these points.

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).