

Newton's Law of Cooling

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Objective

To investigate Newton's Law of Cooling.

Narrative

Newton's Law of Cooling states that the rate dT/dt at which the temperature $T = T(t)$ of an object changes with respect to time t , is proportional to the difference $A - T$ between the ambient temperature A of the environment, and the temperature T of the object; that is

$$\frac{dT}{dt} = k(A - T)$$

where $k > 0$ is a positive real constant.

In this project we investigate Newton's Law of Cooling. We use the command:

DSolve[*differential equation in x and y*, **y**, **x**] solves the differential equation in x and y for $y = y(x)$.

As illustrated below, in using **DSolve** we be careful in specifying y by "**y[x]**".

Tasks

1. Type the command lines below into Mathematica in the order in which they are listed. These commands produce the direction field associated to Newton's Law of Cooling $T' = k(A - T)$ assuming $k = 0.5$ and $A = 80$, and the three solutions corresponding to the initial conditions $T(0) = 10, 60$, and 120 .

```
In[1] := (* Your name, today's date *)
In[2] := (* Newton's Law of Cooling *)
In[3] := DSolve[{T'[t] == 0.5 (80-T[t]), T[0]==10}, T, t]
In[4] := T1[t_] = -70Exp[-0.5t]+80
In[5] := DSolve[{T'[t] == 0.5 (80-T[t]), T[0]==60}, T, t]
In[6] := T2[t_] = -20Exp[-0.5t]+80
In[7] := DSolve[{T'[t] == 0.5 (80-T[t]), T[0]==120}, T, t]
In[8] := T3[t_] = 40Exp[-0.5t]+80
In[9] := Plot[{T1[t], T2[t], T3[t]}, {t,0,4}]
```

At this point, make a hard-copy of your typed input and Mathematica's responses (both text and graphics). Then:

2. Label by hand each of the graphs you created in Task 1: label the graph corresponding to the initial condition $T(0) = 10$ by " $T(0) = 10$ ", for example.
3. Draw and label by hand the line whose equation is $T = 80$.
4. On the last graph you produced in Task 1, plot by hand and label the solutions to the differential equation $T' = 0.5T(80 - T)$ that pass through the points $P(2, 60)$ and $P(2, 100)$.
5. Use the curves you drew in Task 1 to estimate $T(4)$.
6. If $T(0) < A$:
 - a) what does Newton's Law of Cooling imply about the sign of dT/dt ? is T increasing or decreasing?
 - b) explain (on physical grounds) why T should increase to A as t gets large.

7. If $T(0) > A$:

- a) what does Newton's Law of Cooling imply about the sign of dT/dt ? is T increasing or decreasing?
- b) explain (on physical grounds) why T should decrease to A as t gets large.

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-labeled graphics), together with your hand-written responses.