# Ballistics

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### *Objective*

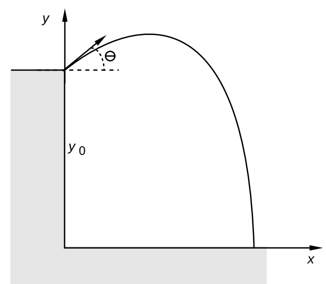
To illustrate the application of parametric curves to ballistics.

### *Narrative*

If a projectile is fired vertically upward into the air with an initial velocity of $v_0$ ft/sec from a point $s_0$ ft above the ground, then (neglecting air resistance) after $t$ sec the projectile is

$$s = s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$$

feet above the ground, where $g = 32$ ft/sec$^2$ is acceleration due to gravity. If the projectile is fired at an angle of elevation of $\theta$ with respect to the horizontal at an initial velocity of $v_0$ ft/sec from a point $y_0$ ft above the ground (see the figure to the right), then (neglecting air resistance) after $t$ sec the projectile is located at the point whose coordinates are given parametrically by




$$x(t) = (v_0 \cos\theta)t, \quad y(t) = -\frac{1}{2}gt^2 + (v_0 \sin\theta)t + y_0.$$

### *Task*

**1**. Type the command lines below into Mathematica in the order in which they are listed. They describe the motion of a projectile that is fired at time $t_0 = 0$ at an angle of elevation of $\theta = \pi/4$ with respect to the horizontal at an initial velocity of $v_0 = 128$ ft/sec from a point $y_0 = 100$ ft above the ground.

```
In[1]:= (* Your name, today´s date *)
In[2]:= (* Ballistics *)
In[3]:= {g=32, t0=0, y0=100, v0=128, Theta=Pi/4}
In[4]:= x[t_] := v0*Cos[Theta]*t
In[5]:= y[t_] := -0.5*g*t^2+v0*Sin[Theta]*t+y0
In[6]:= {vx[t_] := x´[t], vy[t_] := y´[t]}
In[7]:= {t1 = 1, y[t1],
        ParametricPlot[{x[t],y[t]}, {t,t0,t1}, AspectRatio->1, AxesOrigin->{0, 0}]}
```

(The quantities **vx** and **vy** are known as the $x$- and $y$-components of the velocity, respectively. We will discuss these quantities later in our discussion of vectors.)

**2**. By using trial-and-error, change the value of **t1** in the last line you typed until you obtain a value of **t1** *greater than* **t0** for which **y[t1]** is within 2 decimal places of 0. (In doing this you are estimating the time **t1** it takes the projectile to hit the ground.)

**3**. Continue by typing the command lines in the left-hand column below into Mathematica in the order in which they are listed.

| | |
|---|---|
| `In[8]:=` **y[t1]** | This is the range of the projectile. |
| `In[9]:=` **Sqrt[vx[t1]^2+vy[t1]^2]** | This is the impact speed. (We discuss impact speed further in our discussion of vectors.) |
| `In[10]:=` **Solve[v[t]==0,t]** | See below. |
| `In[11]:=` **t /. Solve[v[t]==0,t]** | Again, see below. |
| `In[12]:=` **tmax = %[[1]]** | This is the time at which the vertical component of the velocity is 0; this is the time at which the projectile achieves its maximum altitude. |
| `In[13]:=` **y[tmax]** | This is the maximum altitude. |

The effect of the code, "**Solve[v[t]==0,t]**" in line 10 is to solve the equation $v(t) = 0$ for $t$; the result of this code is a list $L$ containing the rule $t \to 4$ that associates to $t$ the value 4. (If the equation $v(t) = 0$ had more than one solution then $L$ would have more than one entry.) The effect of the code, "**t /. Solve[v[t]==0,t]**" in line 11 is to create a list whose entry is the value obtained by applying the rule contained in $L$ to **t**. (If $L$ had more than one entry then the result of line 11 would be the list obtained by applying each rule to **t**.)

The *range* of a projectile is the horizontal (or $x$-) distance the projectile travels before it strikes the ground. To compute the range of a projectile that is fired at an angle of elevation of $\theta$ with respect to the horizontal at an initial velocity of $v_0$ ft/sec from a point $y_0$ ft above the ground in closed form, observe that at the time $t_1$ the projectile strikes the ground,

$$y(t_1) = -\frac{1}{2}gt_1^2 + (v_0 \sin\theta)t_1 + y_0 = 0.$$

Thus $t_1 = (v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta + 2gy_0})/g$, and the range

$$x(t_1) = (v_0 \cos\theta)t_1 = v_0 \cos\theta \frac{v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta + 2gy_0}}{g}.$$

And if $y_0 = 0$, then the range

$$x(t_1) = \frac{v_0^2 \sin 2\theta}{g}.$$

**4**. What is the maximum range of a projectile fired with initial velocity $v_0$ from ground level $y_0 = 0$? At what angle $\theta$ is this maximum range achieved? Justify your answer.

Your lab report will be a hard copy of your typed input and Mathematica's responses, as well as your written responses.

***Comments***

There are numerous other questions about ballistics that you are now able to handle and that might be of interest to you. For further information, consult your instructor!

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