Ballistics

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Objective

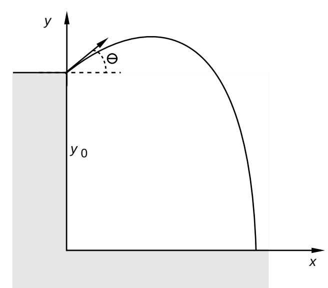
To illustrate the application of parametric curves to ballistics.

Narrative

If a projectile is fired vertically upward into the air with an initial velocity of v_0 m/sec from a point s_0 meters above the ground, then (neglecting air resistance) after t sec the projectile is

$$s = s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$$

meters above the ground, where $g = 9.8$ m/sec^2 is acceleration due to gravity. If the projectile is fired at an angle of elevation of θ with respect to the horizontal at an initial velocity of v_0 m/sec from a point y_0 meters above the ground (see the figure to the right), then (neglecting air resistance) after t sec the projectile is located at the point whose coordinates are given parametrically by



$$x(t) = (v_0 \cos\theta)t, \quad y(t) = -\frac{1}{2}gt^2 + (v_0 \sin\theta)t + y_0.$$

Task

1. Type the command lines below into Mathematica in the order in which they are listed. They describe the motion of a projectile that is fired at time $t_0 = 0$ at an angle of elevation of $\theta = \pi/4$ with respect to the horizontal at an initial velocity of $v_0 = 128$ m/sec from a point $y_0 = 100$ meters above the ground.

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In[1]:= (* Your name, today´s date *)
In[2]:= (* Ballistics *)
In[3]:= {g=9.8, t0=0, y0=100, v0=128, Theta=Pi/4}
In[4]:= x[t_] := v0*Cos[Theta]*t
In[5]:= y[t_] := -0.5*g*t^2+v0*Sin[Theta]*t+y0
In[6]:= {vx[t_] := x´[t], vy[t_] := y´[t]}
In[7]:= {t1 = 1, y[t1],
          ParametricPlot[{x[t],y[t]}, {t,t0,t1}, AspectRatio->1, AxesOrigin->{0, 0}]}
```

(The quantities **vx** and **vy** are known as the x- and y-components of the velocity, respectively. We will discuss these quantities later in our discussion of vectors.)

2. By using trial-and-error, change the value of **t1** in the last line you typed until you obtain a value of **t1** *greater than* **t0** for which **y[t1]** is within 2 decimal places of 0. (In doing this you are estimating the time **t1** it takes the projectile to hit the ground.)

3. Continue by typing the command lines in the left-hand column below into Mathematica in the order in which they are listed.

`In[8]:=` **y[t1]**	This is the range of the projectile.
`In[9]:=` **Sqrt[vx[t1]^2+vy[t1]^2]**	This is the impact speed. (We discuss impact speed further in our discussion of vectors.)
`In[10]:=` **Solve[v[t]==0,t]**	See below.
`In[11]:=` **t /. Solve[v[t]==0,t]**	Again, see below.
`In[12]:=` **tmax = %[[1]]**	This is the time at which the vertical component of the velocity is 0; this is the time at which the projectile achieves its maximum altitude.
`In[13]:=` **y[tmax]**	This is the maximum altitude.

The effect of the code, "**Solve[v[t]==0,t]**" in line 10 is to solve the equation $v(t) = 0$ for t; the result of this code is a list L containing the rule $t \to 4$ that associates to t the value 4. (If the equation $v(t) = 0$ had more than one solution then L would have more than one entry.) The effect of the code, "**t /. Solve[v[t]==0,t]**" in line 11 is to create a list whose entry is the value obtained by applying the rule contained in L to **t**. (If L had more than one entry then the result of line 11 would be the list obtained by applying each rule to **t**.)

The *range* of a projectile is the horizontal (or x-) distance the projectile travels before it strikes the ground. To compute the range of a projectile that is fired at an angle of elevation of θ with respect to the horizontal at an initial velocity of v_0 m/sec from a point y_0 meters above the ground in closed form, observe that at the time t_1 the projectile strikes the ground,

$$y(t_1) = -\frac{1}{2}gt_1^2 + (v_0 \sin\theta)t_1 + y_0 = 0.$$

Thus $t_1 = (v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta + 2gy_0})/g$, and the range

$$x(t_1) = (v_0 \cos\theta)t_1 = v_0 \cos\theta \frac{v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta + 2gy_0}}{g}.$$

And if $y_0 = 0$, then the range

$$x(t_1) = \frac{v_0^2 \sin 2\theta}{g}.$$

4. What is the maximum range of a projectile fired with initial velocity v_0 from ground level $y_0 = 0$? At what angle θ is this maximum range achieved? Justify your answer.

Your lab report will be a hard copy of your typed input and Mathematica's responses, as well as your written responses.

Comments

There are numerous other questions about ballistics that you are now able to handle and that might be of interest to you. For further information, consult your instructor!