Sequences and Series: Part 1

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Objective

To investigate sequences and series using Mathematica.

Narrative

In this project we investigate:

1. the sequence $\{a_n\} = \left\{\frac{1}{2^n}\right\}$ and the associated series $\Sigma_{n=1}^{\infty}\frac{1}{2^n}$,
2. the sequence $\{a_n\} = \left\{\frac{1}{n(n+1)}\right\}$ and the associated series $\Sigma_{n=1}^{\infty}\frac{1}{n(n+1)}$, and
3. the sequence $\{a_n\} = \left\{\frac{n}{n+1}\right\}$ and the associated series $\Sigma_{n=1}^{\infty}\frac{n}{n+1}$.

Tasks

1. Type the command lines in the left-hand column below into Mathematica in the order in which they are listed. These commands initialize this project.

```
In[1]:= (* Your name, today's date *)
In[2]:= (* Sequences and Series: Part 1 *)
In[3]:= <<Graphics`MultipleListPlot`
```

2. a) Type the command lines below into Mathematica in the order in which they are listed. These commands list some terms in the sequence $\{a_n\} = \left\{\frac{1}{2^n}\right\}$, find $\lim_{n\to\infty} a_n$, list some terms in the sequence of partial sums $\Sigma_{n=1}^{N}\frac{1}{2^n}$, find the series $\Sigma_{n=1}^{\infty}\frac{1}{2^n}$, and graph the original sequence and the sequence of partial sums.

Observe that all the commands below are part of just one (big) command block: at the end of each line (except the last), hit the "Return" key rather than hitting the "Shift+Enter" combination as you would normally do; at the end of the last line only, hit the "Shift+Enter" combination. The reason for doing this is that in Tasks 2(b) and 2(c) you will be asked to reuse the code below for two more sequences, and by typing all the code below in just one (big) command block, you can simply copy the command block below, edit the first line, and then execute the command block by hitting the "Shift+Enter" combination (thereby avoiding a lot of retyping).

```
In[4]:= a[n_] = 1/2^n
        Print["The sequence is"]
        TheSequence = Table[a[n], {n,1,6}]
        Print["The limit of the sequence is"]
        Limit[a[n], n->Infinity]
        Print["The sequence of partial sums is"]
        TheSequenceOfPartialSums = Table[Sum[a[n], {n,1,N}], {N,1,6}]
        Print["The series is"]
        Limit[Sum[a[n], {n,1,N}], N->Infinity]
        Print["The graphs"]
        MultipleListPlot[TheSequence, TheSequenceOfPartialSums]
```

b) Repeat part (a) using the sequence $\{a_n\} = \left\{\frac{1}{n(n+1)}\right\}$.

c) Repeat part (a) using the sequence $\{a_n\} = \left\{ \dfrac{n}{n+1} \right\}$.

At this time, make a hard-copy of your typed input and Mathematica's responses. Then:

3. For each of the (three) parts of Task 2:

a) connect the dots in the graph of the original sequence $\{a_n\}$, and the dots in the graph of the sequence of partial sums $\{s_n\}$ by hand,

b) label the sequence $\{a_n\}$ as "$\{a_n\}$", and the sequence $\{s_n\}$ as "$\{s_n\}$" by hand,

c) if the sequence $\{a_n\}$ converges to L, draw the horizontal line $y = L$ and write "The sequence $\{a_n\}$ converges to ___." next to it, filling in the blank with the number L; if the sequence $\{a_n\}$ diverges, write "The sequence $\{a_n\}$ diverges." next to its graph, and

d) if the sequence $\{s_n\}$ converges to L, draw the horizontal line $y = L$ and write "The series $\Sigma_{n=1}^{\infty} a_n$ converges to ___." next to it, filling in the blank with the number L; if the sequence $\{s_n\}$ diverges, write "The series $\Sigma_{n=1}^{\infty} a_n$ diverges." next to its graph.

Your lab report will be a hard-copy of your typed input and Mathematica's responses (both text and hand-labeled graphics).