

Maclaurin and Taylor Series

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Objective

To investigate the approximation of functions by Maclaurin and Taylor series using Mathematica.

Narrative

In this project we investigate the approximation of $f(x) = \sin x$ by its Maclaurin series, and $f(x) = \ln x$ by its Taylor series. We do this by using the command “**Series**[$f[x]$, { x,c,n }]” to generate the n th order Taylor series for $f(x)$ about the point $x = c$. (The n th order Maclaurin series for $f(x)$ is just the n th order Taylor series for $f(x)$ about the point $x = 0$.)

Tasks

1. Let us begin by illustrating our usage of the **Series** command. Type the command lines in the left-hand column below into Mathematica in the order in which they are listed. Line 4 illustrates the syntax of the **Series** command. Line 5 illustrates that the **Normal** command truncates a Maclaurin (or Taylor) series expansion. And line 6 illustrates how we can compose the **Series** and **Normal** commands.

```
In[1] := (* Your name, today's date *)
In[2] := (* Maclaurin and Taylor Series *)
In[3] := (* Task 1 *)
In[4] := Series[Exp[x], {x,0,3}]
In[5] := Normal[%]
In[6] := Normal[Series[Exp[x], {x,0,3}]]
```

2. a) Type the command lines below into Mathematica in the order in which they are listed. These commands produce several Maclaurin series approximations to $f(x) = \sin x$, and graph $f(x)$ and these approximations on one set of coordinate axes.

```
In[7] := (* Task 2a *)
In[8] := p1 = Normal[Series[Sin[x], {x,0,1}]]
In[9] := p3 = Normal[Series[Sin[x], {x,0,3}]]
In[10] := p5 = Normal[Series[Sin[x], {x,0,5}]]
In[11] := p7 = Normal[Series[Sin[x], {x,0,7}]]
In[12] := p9 = Normal[Series[Sin[x], {x,0,9}]]
In[14] := Plot[{Sin[x],p1,p3,p5,p7,p9}, {x,0,2Pi}]
```

b) Continue by typing the command lines below into Mathematica in the order in which they are listed. These commands produce several Taylor series approximations to $f(x) = \ln x$ about $c = 1$, and graph $f(x)$ and these approximations on one set of coordinate axes.

```
In[13] := (* Task 2b *)
In[15] := p1 = Normal[Series[Log[x], {x,1,1}]]
In[16] := p2 = Normal[Series[Log[x], {x,1,2}]]
In[17] := p3 = Normal[Series[Log[x], {x,1,3}]]
In[18] := p4 = Normal[Series[Log[x], {x,1,4}]]
In[19] := p5 = Normal[Series[Log[x], {x,1,5}]]
In[20] := Plot[{Log[x],p1,p2,p3,p4,p5}, {x,0,2Pi}]
```

At this time, make a hard-copy of your typed input and Mathematica's responses. Then:

2. On the graphic you created in Task 2a, label by hand the graphs of $f(x)$ and $p_1(x)$, $p_3(x)$, $p_5(x)$, $p_7(x)$, and $p_9(x)$. (Label the graph of $p_1(x)$ by “ $p_1(x)$ ”, for example.)

3. On the graphic you created in Task 2b:

- a) draw the line whose equation is $x = -1$ by hand, and
- b) label by hand the graphs of $f(x)$ and $p_1(x)$, $p_2(x)$, $p_3(x)$, $p_4(x)$, and $p_5(x)$. (Label the graph of $p_1(x)$ by “ $p_1(x)$ ”, for example.)

Your lab report will be a hard-copy of your typed input and Mathematica’s responses (both text and hand-labeled graphics).

Comments

Note how well both power series approximate the given functions. Also note the differences between power series which converge over the entire real number line ($f(x) = \sin x$) and those that do not ($f(x) = \ln x$).